Jom Skor A+ SPM 2021

MATEMATIK TAMBAHAN

Integration

18 October 2021 | Monday

8.00 pm - 10.00 pm

https://bit.ly/CgFauziIntegrations

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MRSM Kepala Batas

Anjuran Unit Matematik
Bahagian Pendidikan Menengah

بسم الله الرحمن الرحيم

INTEGRATION

Chapter 3 | Form 5

Mohamad Fauzi Razak

Part 1

Indefinite integral

Definite integral

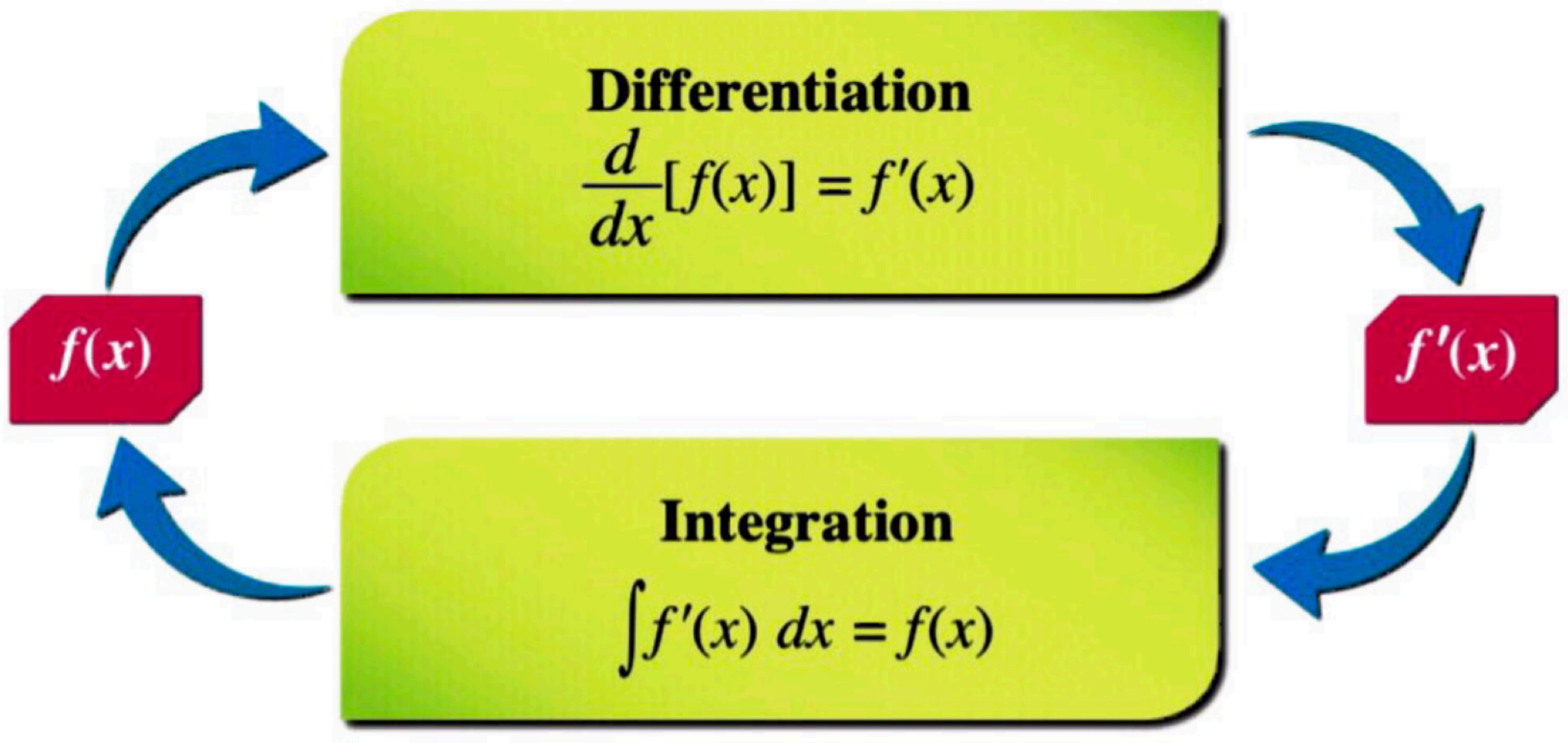
Equation of a curve

Area under a curve

Generated volume

$$y = 4x^3$$

$$\frac{dy}{dx} = 12x^2$$



$$\frac{d(A)}{dx} = B$$

$$\therefore \int B\; dx = A$$

In general,

> If $\frac{d}{dx}[f(x)] = f'(x)$, then the integral of $f'(x)$ with respect to x is $\int f'(x)\, dx = f(x)$.

1. Given $\frac{d}{dx}(3x^2) = 6x$, find $\int 6x\, dx$.

$$\therefore \int 6x\, dx = 3x^2$$

2. Given $y = 3(2x+2)^3$, find $\frac{dy}{dx}$. Subsequently, find $\int [18(2x+2)^2] dx$.

$$\frac{dy}{dx} = 9(2x+2)^2(2)$$
$$= 18(2x+2)^2$$

$$\therefore \int 18(2x+2)^2\, dx$$
$$= 3(2x+2)^3$$

Case 1

$y = 5x$, $\frac{dy}{dx} = 5$ and

$\int 5\ dx = 5x$

Case 2

$y = 5x + 2$, $\frac{dy}{dx} = 5$ and

$\int 5\ dx = 5x + 2$

Case 3

$y = 5x - 3$, $\frac{dy}{dx} = 5$ and

$\int 5\ dx = 5x - 3$

Notice that differentiating those three cases give the same value of $\frac{dy}{dx}$, even though each of them has a different constant. This constant is known as the **constant of integration** and represented by the symbol c. The constant c is added as a part of indefinite integral for a function. For example, $\int 5\ dx = 5x + c$.

y	$\frac{dy}{dx}$
$5x$	5
$5x + 2$	5
$5x - 3$	5

$$\int 5\ dx = 5x + 0$$

$$\int 5\ dx = 5x + 2$$

$$\int 5\ dx = 5x - 3$$

$$\therefore \int 5\ dx = 5x + c$$

Indefinite integral

- $\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c, \; n \neq -1$
- $\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$
- $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \; n \neq -1$

For a constant a,

$\int a \, dx = ax + c$, where a and c are constants.

3. Find the indefinite integral for each of the following.

(a) $\int 12 \, dx$

$$= \int 12 \; x^0 \, dx$$

$$= 12\left(\frac{x^{0+1}}{0+1}\right) + c$$

$$= 12x + c$$

(b) $\int dx$

$$= \int x^0 \, dx$$

$$= \left(\frac{x^{0+1}}{0+1}\right) + c$$

$$= x + c$$

(c) $\int 0 \, dx$

$$= \int 0 \, x^0 \, dx$$

$$= 0\left(\frac{x^{0+1}}{0+1}\right) + c$$

$$= 0x + c$$

$$= c$$

$$\int ax^n\, dx = \frac{ax^{n+1}}{n+1} + c,\ n \neq -1$$

4. Find the indefinite integral for each of the following.

(a) $\int x^3\, dx$

$$= \frac{x^{3+1}}{3+1} + c$$

$$= \frac{x^4}{4} + c$$

(b) $\int \frac{2}{3x^2}\, dx$

$$= \frac{2}{3}\int x^{-2}\, dx$$

$$= \frac{2}{3}\left(\frac{x^{-2+1}}{-2+1}\right) + c$$

$$= \frac{2}{3}\left(\frac{x^{-1}}{-1}\right) + c$$

$$= -\frac{2}{3x} + c$$

$$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

(c) $\int (4x^2 + 5x)\, dx$

$$= \frac{4\, x^{2+1}}{2+1} + \frac{5\, x^{1+1}}{1+1} + c$$

$$= \frac{4x^3}{3} + \frac{5x^2}{2} + c$$

(d) $\int (x+2)(x-y)\, dx$

$$= \int \left(x^2 - xy + 2x - 2y\right) dx$$

$$= \frac{x^{2+1}}{2+1} - \frac{x^{1+1}y}{1+1} + \frac{2x^{1+1}}{1+1} - \frac{2yx^{0+1}}{0+1} + c$$

$$= \frac{x^3}{3} - \frac{x^2y}{2} + \frac{2x^2}{2} - 2yx + c$$

$$= \frac{x^3}{3} - \frac{x^2y}{2} + x^2 - 2yx + c$$

5. Find the indefinite integral for each of the following.

(a) $\int (3x-5)^9\,dx$

By using sub. method (chain-rule):

$$let\ u = 3x-5 \qquad \frac{du}{dx} = 3$$

$$dx = \frac{1}{3}\,du$$

$$\int (3x-5)^9\,dx = \int u^9\,dx$$

$$= \frac{1}{3}\int u^9\,du$$

$$= \frac{1}{3}\left(\frac{u^{10}}{10}\right) + c$$

$$= \frac{u^{10}}{30} + c$$

$$\therefore \; = \frac{(3x-5)^{10}}{30} + c$$

By using formula: $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$

$$\int (3x-5)^9\,dx = \frac{(3x-5)^{10}}{10(3)} + c$$

$$= \frac{(3x-5)^{10}}{30} + c$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$$

5. Find the indefinite integral for each of the following.

(b) $\displaystyle\int \frac{12}{(2x-6)^3}\,dx$

$$= \int 12\left(2x-6\right)^{-3}\,dx$$

$$= \frac{12\left(2x-6\right)^{-2}}{-2(2)} + c$$

$$= -\,3\left(2x-6\right)^{-2} + c$$

$$= -\,\frac{3}{\left(2x-6\right)^2} + c$$

$$= -\,\frac{3}{\left(2(x-3)\right)^2} + c$$

$$= -\,\frac{3}{4(x-3)^2} + c$$

6. The gradient function of a curve at point (x, y) is given by $\frac{dy}{dx} = 15x^2 + 4x - 3$. If the curve passes through the point $(-1,2)$, find the equation of the curve.

Equation of a curve

Given a gradient function $\frac{dy}{dx} = f'(x)$, then the equation of the curve for the function is $y = \int f'(x)\, dx$.

$$\frac{dy}{dx} = 15x^2 + 4x - 3$$

$$y = \int \left(15x^2 + 4x - 3\right) dx$$

$$y = \frac{15x^3}{3} + \frac{4x^2}{2} - 3x + c$$

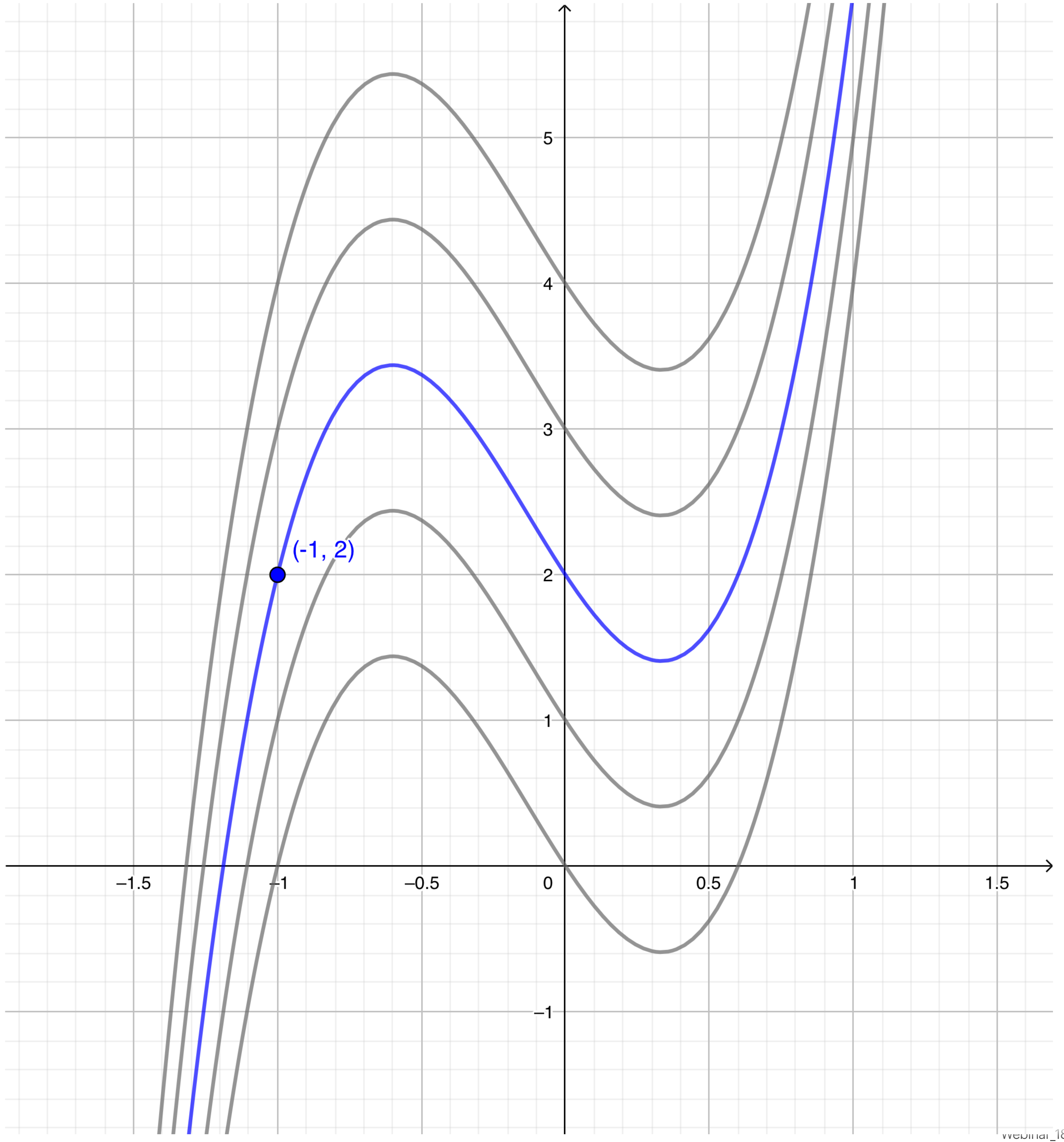
$$y = 5x^3 + 2x^2 - 3x + c\,;(-1, 2)$$

$$2 = 5(-1)^3 + 2(-1)^2 - 3(-1) + c$$

$$2 = -5 + 2 + 3 + c$$

$$c = 2$$

$$\therefore y = 5x^3 + 2x^2 - 3x + 2$$



7. It is given that the gradient of a normal to a curve at one point is $\dfrac{1}{6x-2}$. If the curve passes through point (2, 2), find the equation for that curve.

Equation of a curve

Given a gradient function $\dfrac{dy}{dx} = f'(x)$, then the equation of the curve for the function is $y = \int f'(x)\, dx$.

$$M_N \times M_t = -1$$

$$\frac{1}{6x-2} \times \frac{dy}{dx} = -1$$

$$\therefore \frac{dy}{dx} = -(6x-2)$$

$$= -6x+2$$

$$y = \int (-6x+2)\, dx$$

$$y = -\frac{6x^2}{2} + 2x + c$$

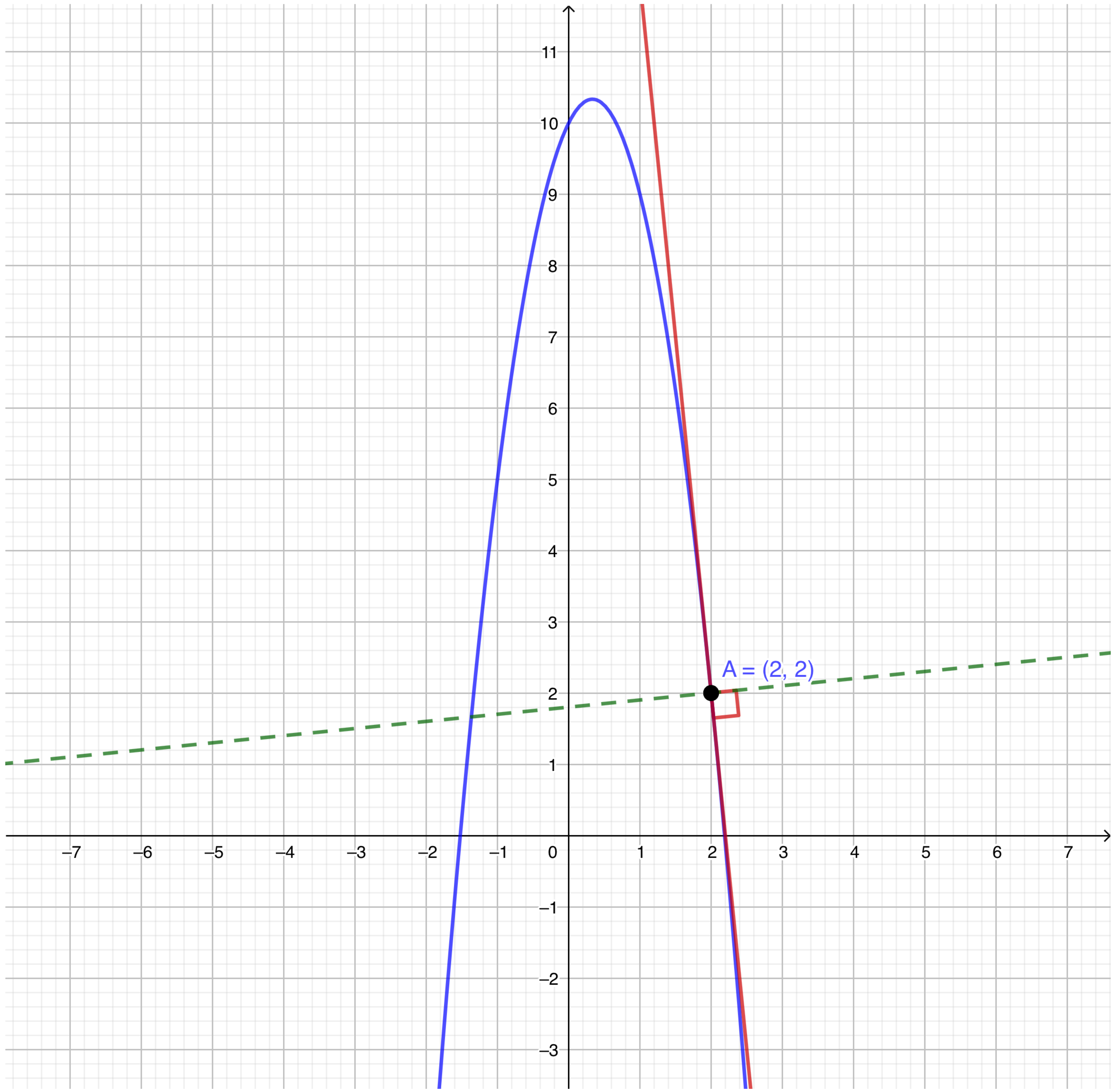
$$y = -3x^2 + 2x + c \;;\; A(2, 2)$$

$$2 = -3(2)^2 + 2(2) + c$$

$$2 = -12 + 4 + c$$

$$c = 10$$

$$\therefore y = -3x^2 + 2x + 10$$



Definite integral

- $\int_a^b f(x)\,dx = \left[g(x)+c\right]_a^b = g(b) - g(a)$

8. Find the value for each of the following definite integrals.

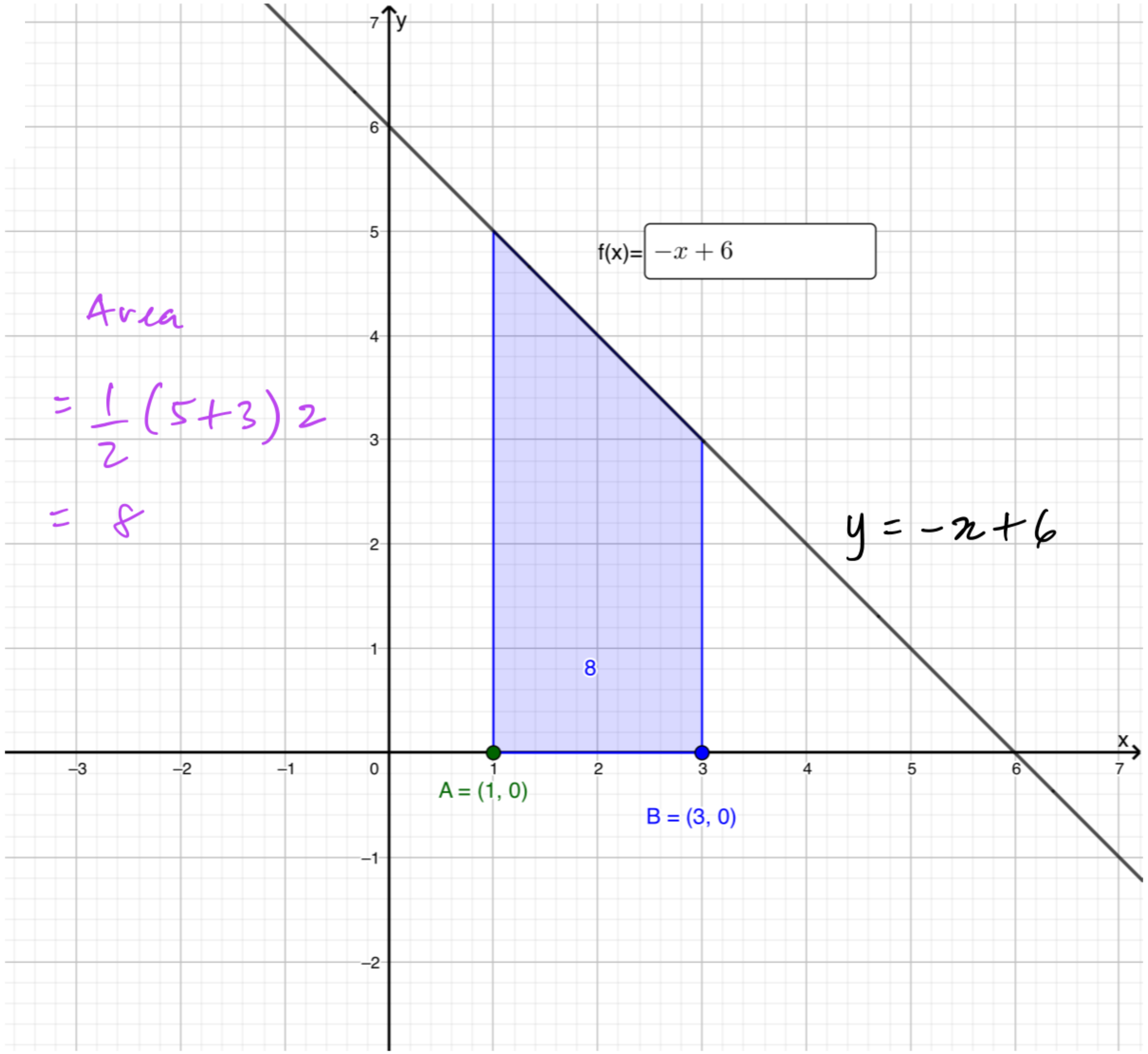
(a) $\int_1^3 (-x+6)\,dx$

$$= \left[-\frac{x^2}{2} + 6x\right]_1^3$$

$$= \left(-\frac{(3)^2}{2} + 6(3)\right) - \left(-\frac{(1)^2}{2} + 6(1)\right)$$

$$= \frac{27}{2} - \frac{11}{2}$$

$$= 8$$

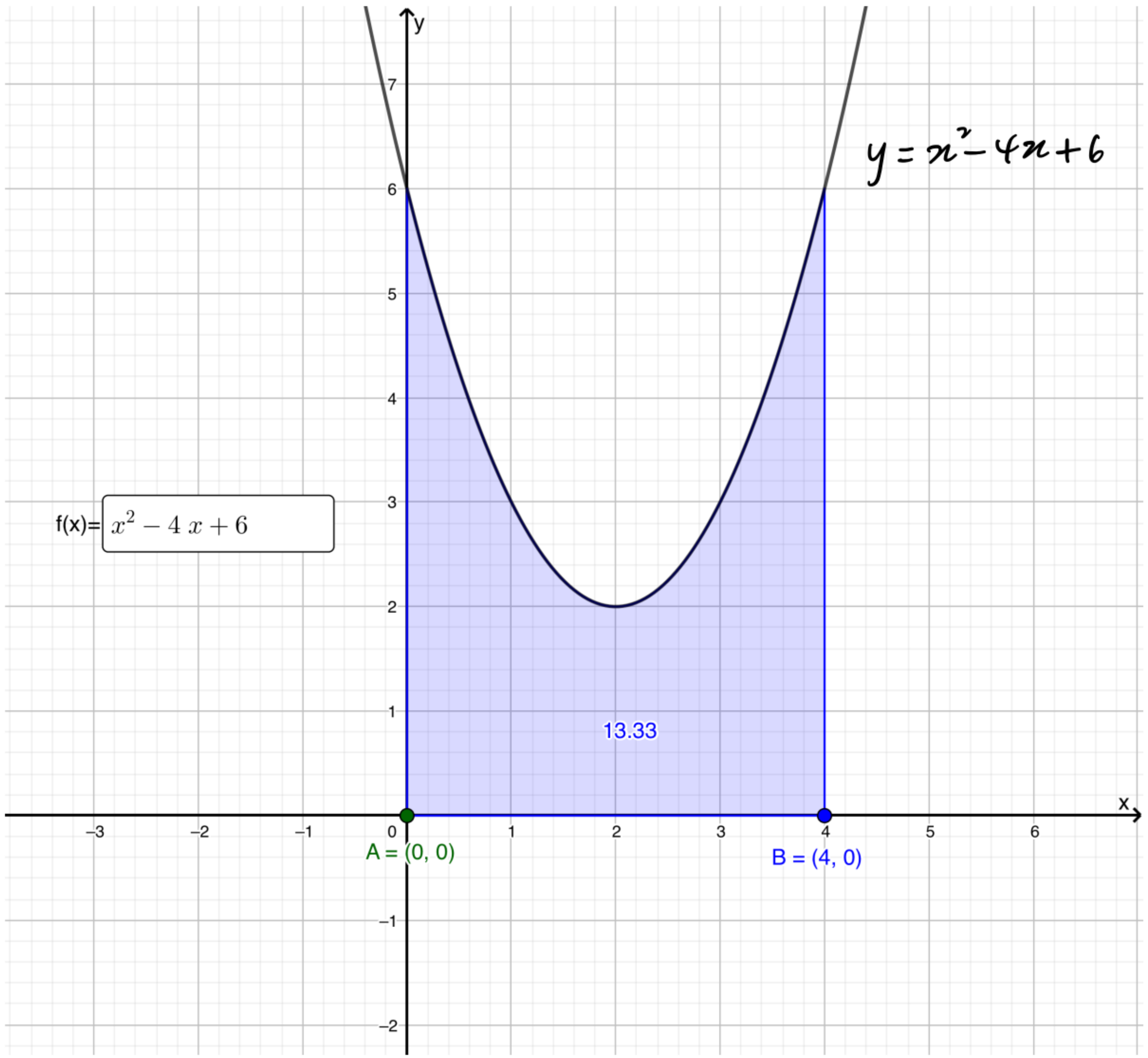


(b) $\int_0^4 (x^2 - 4x + 6)\, dx$

$$= \left[\frac{x^3}{3} - \frac{4x^2}{2} + 6x\right]_0^4$$

$$= \left(\frac{64}{3} - 32 + 24\right) - 0$$

$$= \frac{40}{3} \text{ @ } 13.33$$



- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

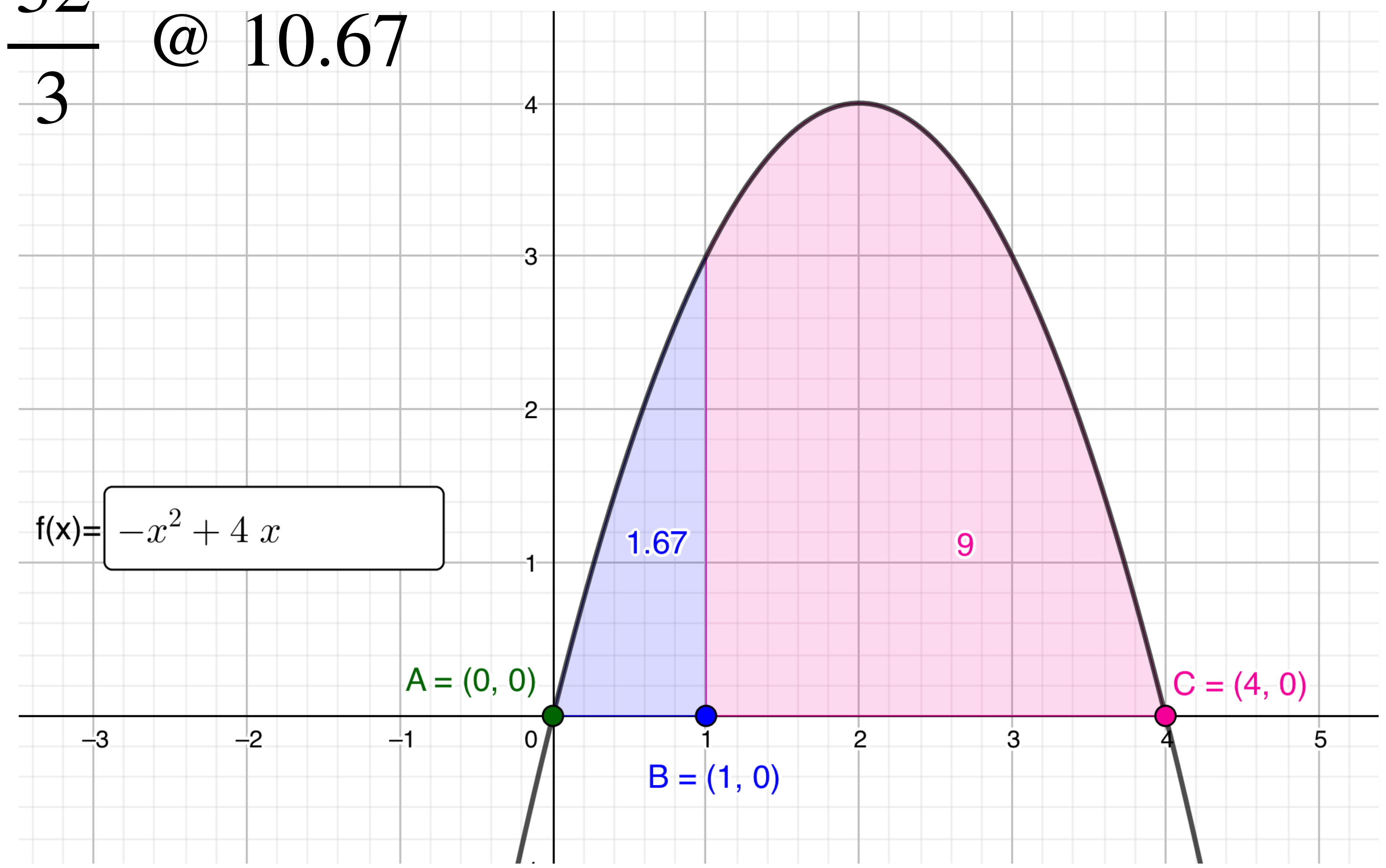
(c) $\int_0^1 (-x^2+4x)\,dx + \int_1^4 (-x^2+4x)\,dx$

$$= \left[-\frac{x^3}{3}+\frac{4x^2}{2}\right]_0^1 + \left[-\frac{x^3}{3}+\frac{4x^2}{2}\right]_1^4$$

$$= \left[\left(-\frac{1}{3}+2\right)-0\right] + \left[\left(-\frac{64}{3}+32\right)-\left(-\frac{1}{3}+2\right)\right]$$

$$= \frac{5}{3}+9$$

$$= \frac{32}{3} \quad @ \ 10.67$$

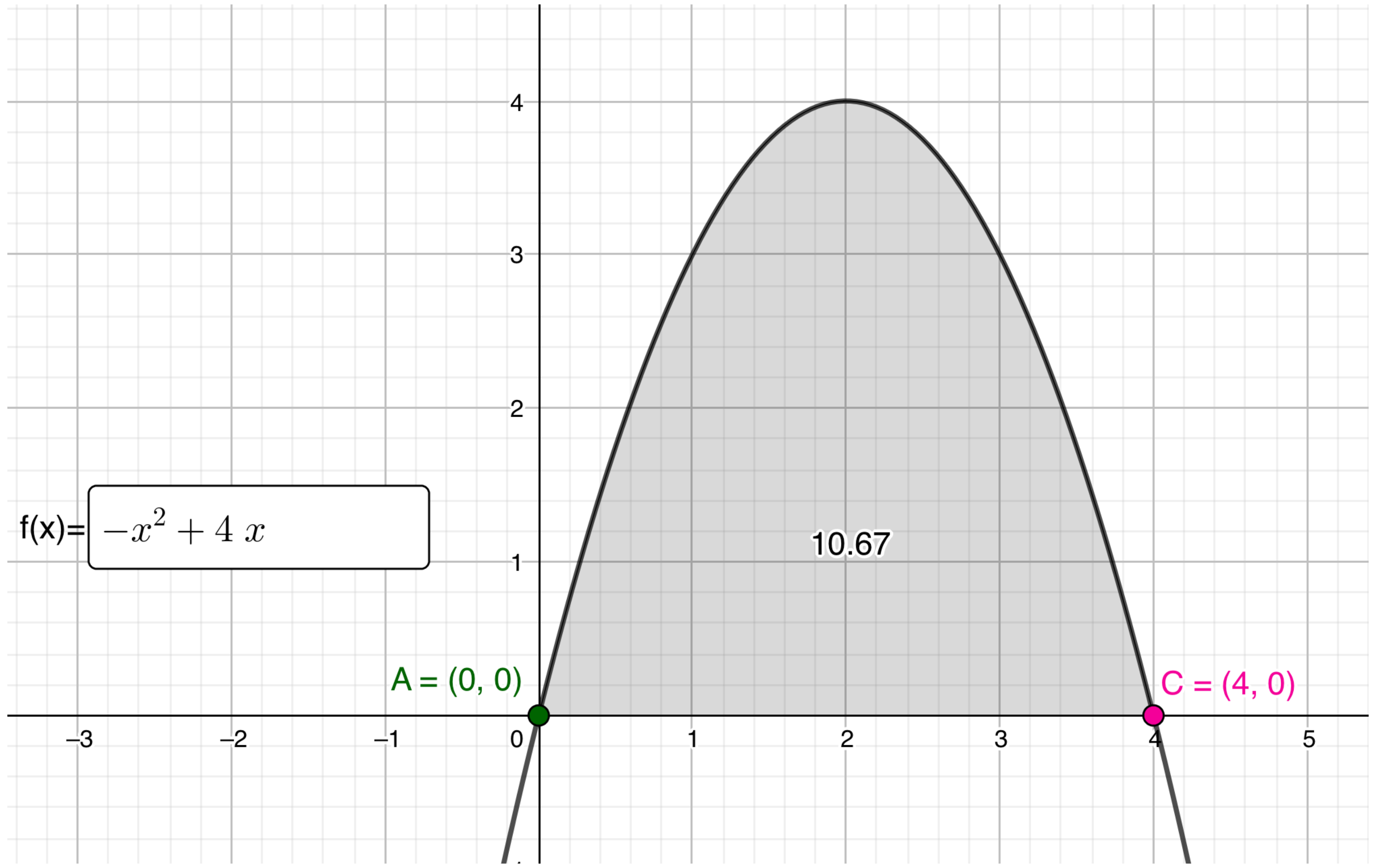


(d) $\int_0^4 (-x^2+4x)\,dx$

$$= \left[-\frac{x^3}{3}+\frac{4x^2}{2}\right]_0^4$$

$$= \left(-\frac{64}{3}+32\right)-0$$

$$= \frac{32}{3} \quad @ \ 10.67$$



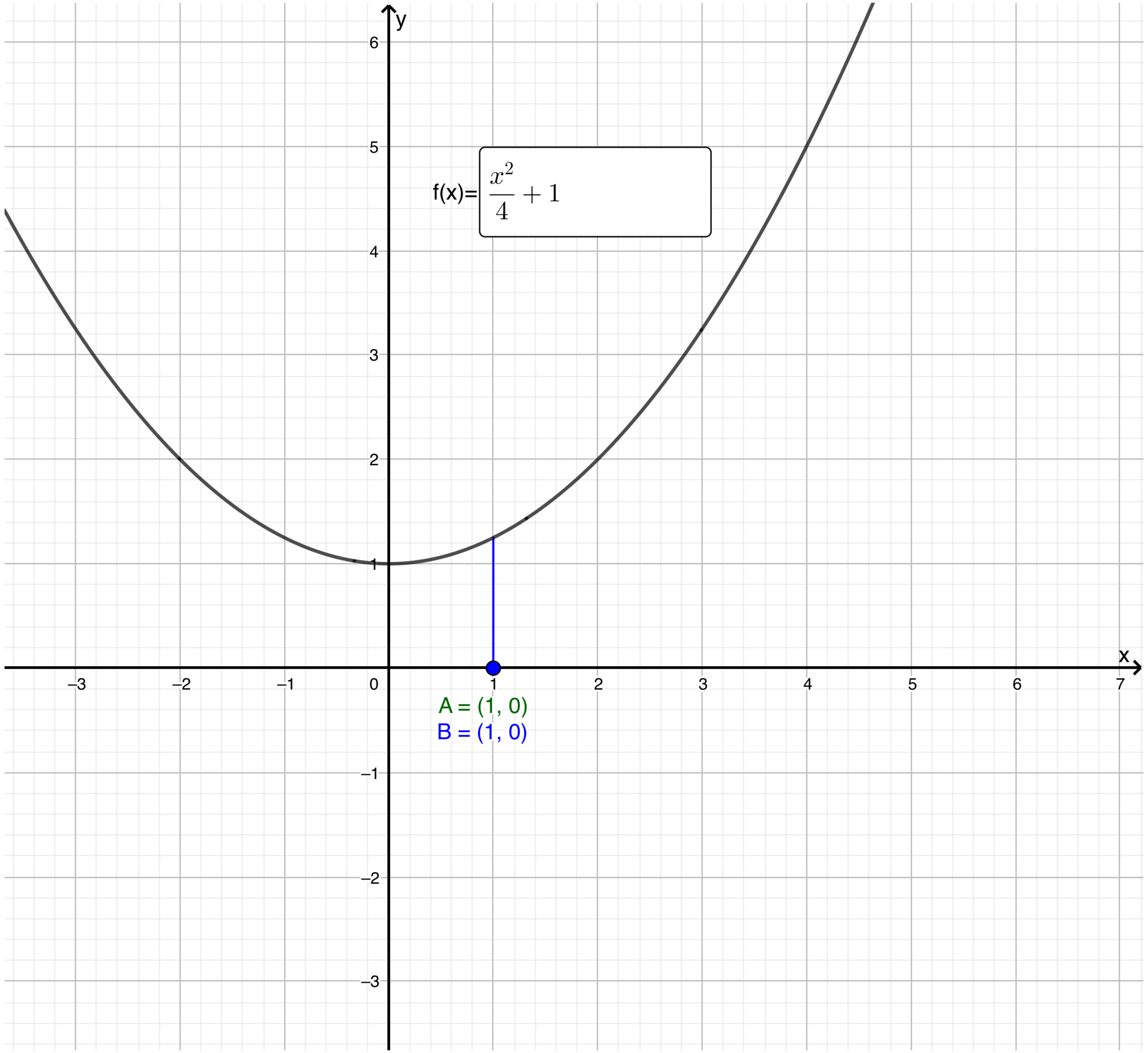
- $\int_a^a f(x)\,dx = 0$

(e) $\int_1^1 \left(\frac{x^2}{4} + 1\right) dx$

$$= \left[\frac{x^3}{3(4)} + x\right]_1^1$$

$$= \left(\frac{1}{12} + 1\right) - \left(\frac{1}{12} + 1\right)$$

$$= 0$$



- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(f) $\int_1^4 x^3\,dx$

$$= \left[\frac{x^4}{4}\right]_1^4$$

$$= \frac{256}{4} - \frac{1}{4}$$

$$= \frac{255}{4} \quad @ \quad 63.75$$

(g) $\int_4^1 x^3\,dx$

$$= \left[\frac{x^4}{4}\right]_4^1$$

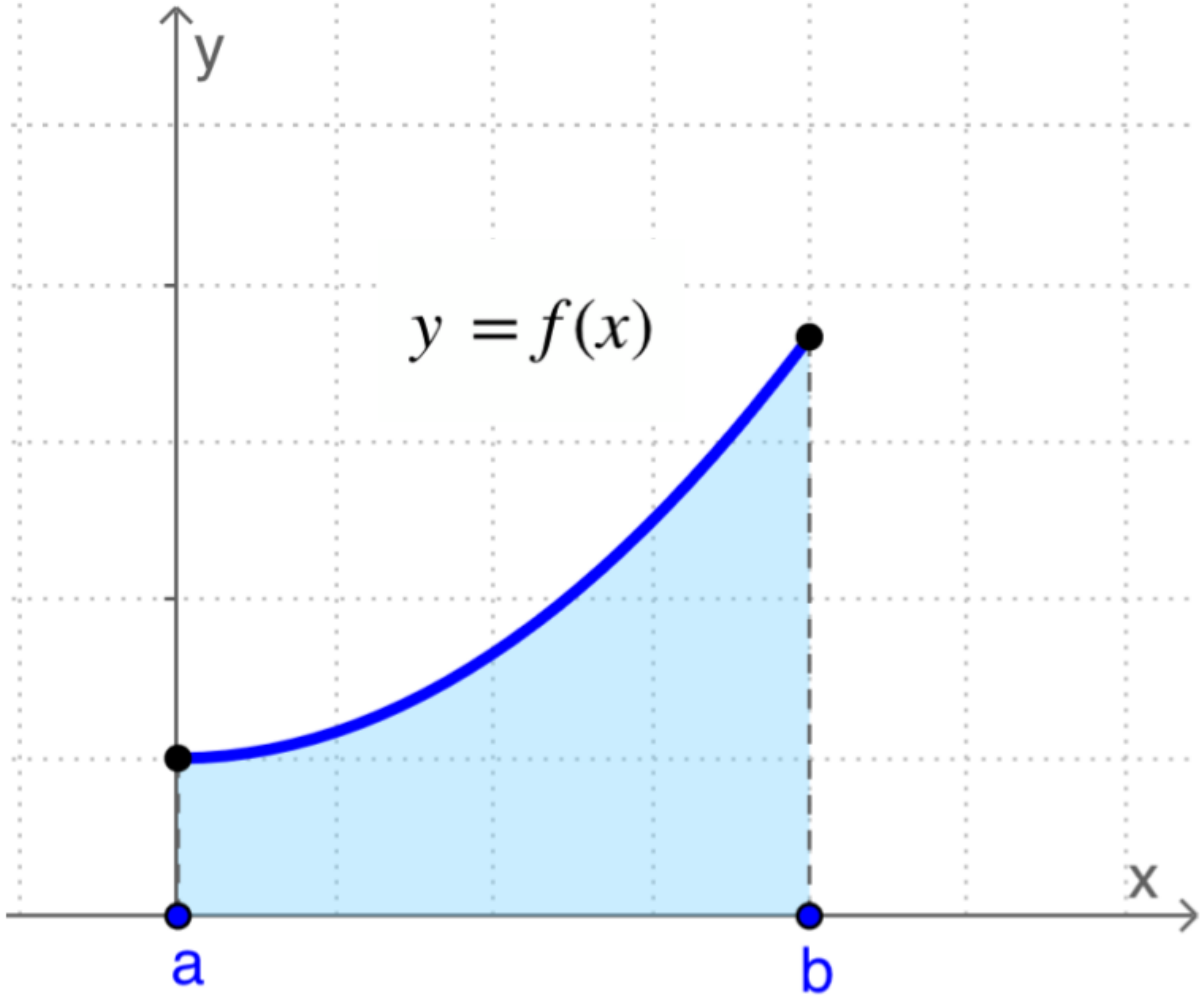
$$= \frac{1}{4} - \frac{256}{4}$$

$$= -\frac{255}{4} \quad @ \quad -63.75$$

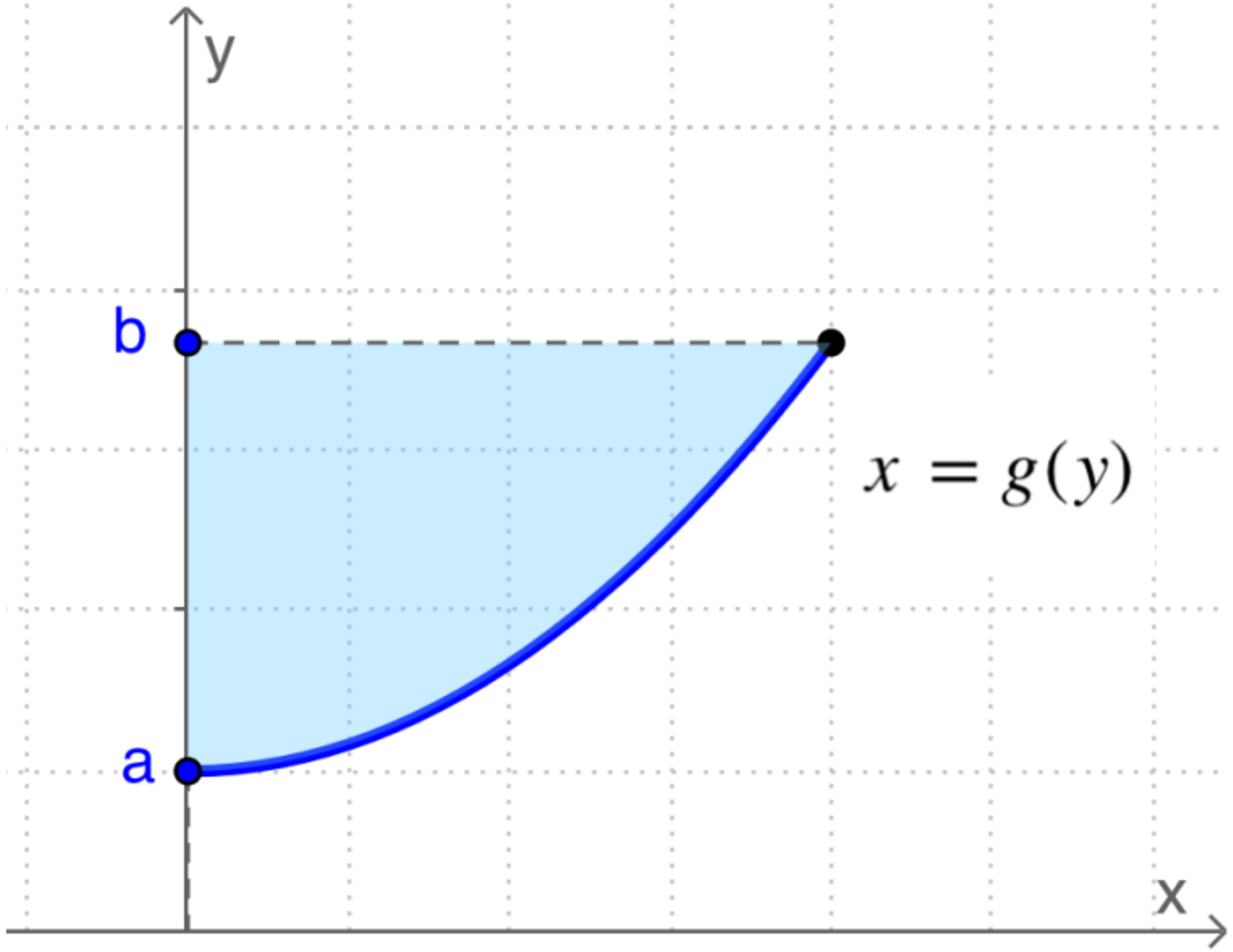
$$\therefore \int_1^4 x^3\,dx = -\int_4^1 x^3\,dx$$

Shown !!!

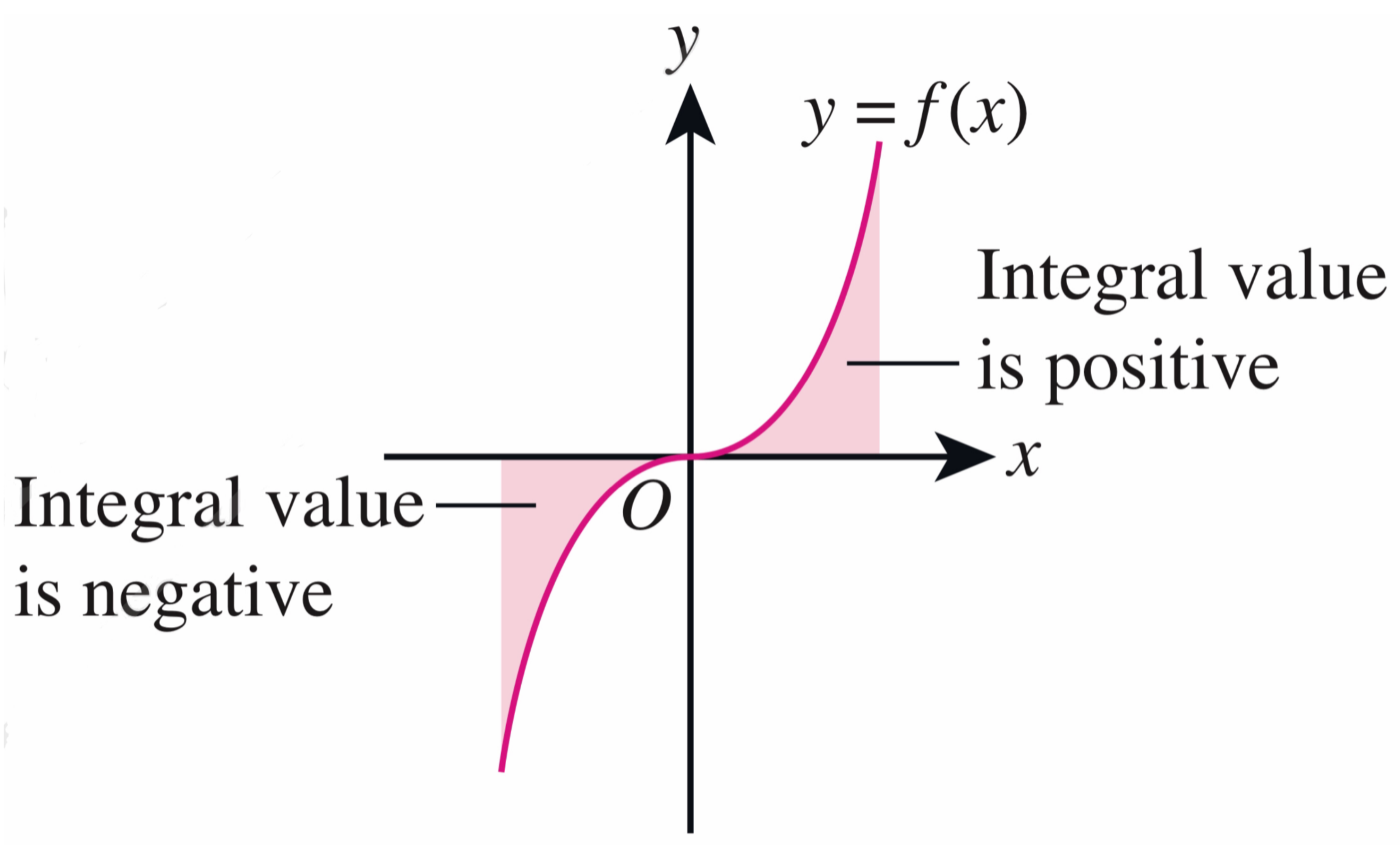
Area under curve



Area of region $= \int_a^b f(x)\,dx$

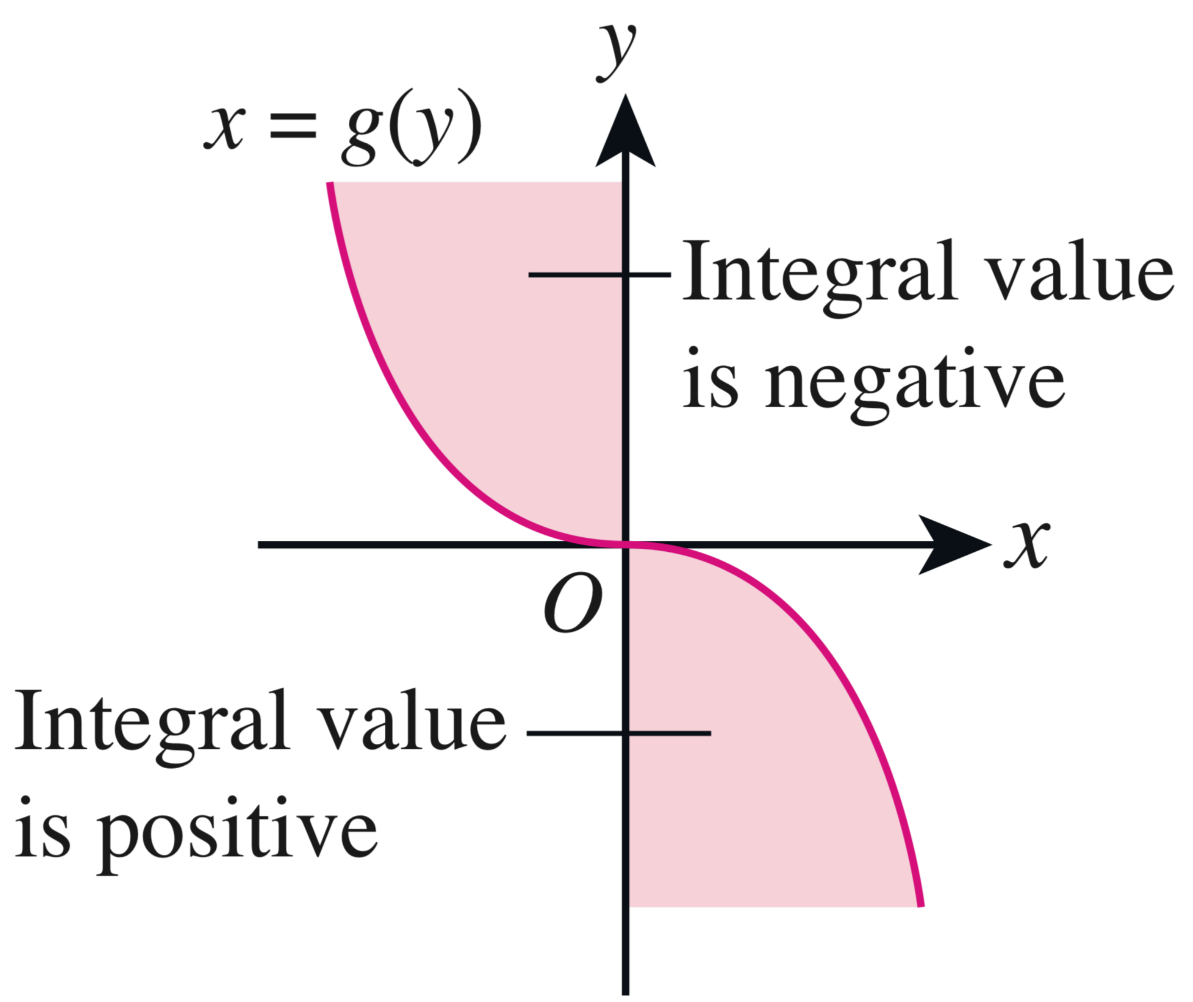


Area of region $= \int_a^b g(y)\,dy$



Area of region
$$= \int_a^b y\,dx$$

For the value of the area bounded by the curve and the x-axis,
- If the region is below the x-axis, then the integral value is negative.
- If the region is above the x-axis, then the integral value is positive.
- The areas of both regions are positive.



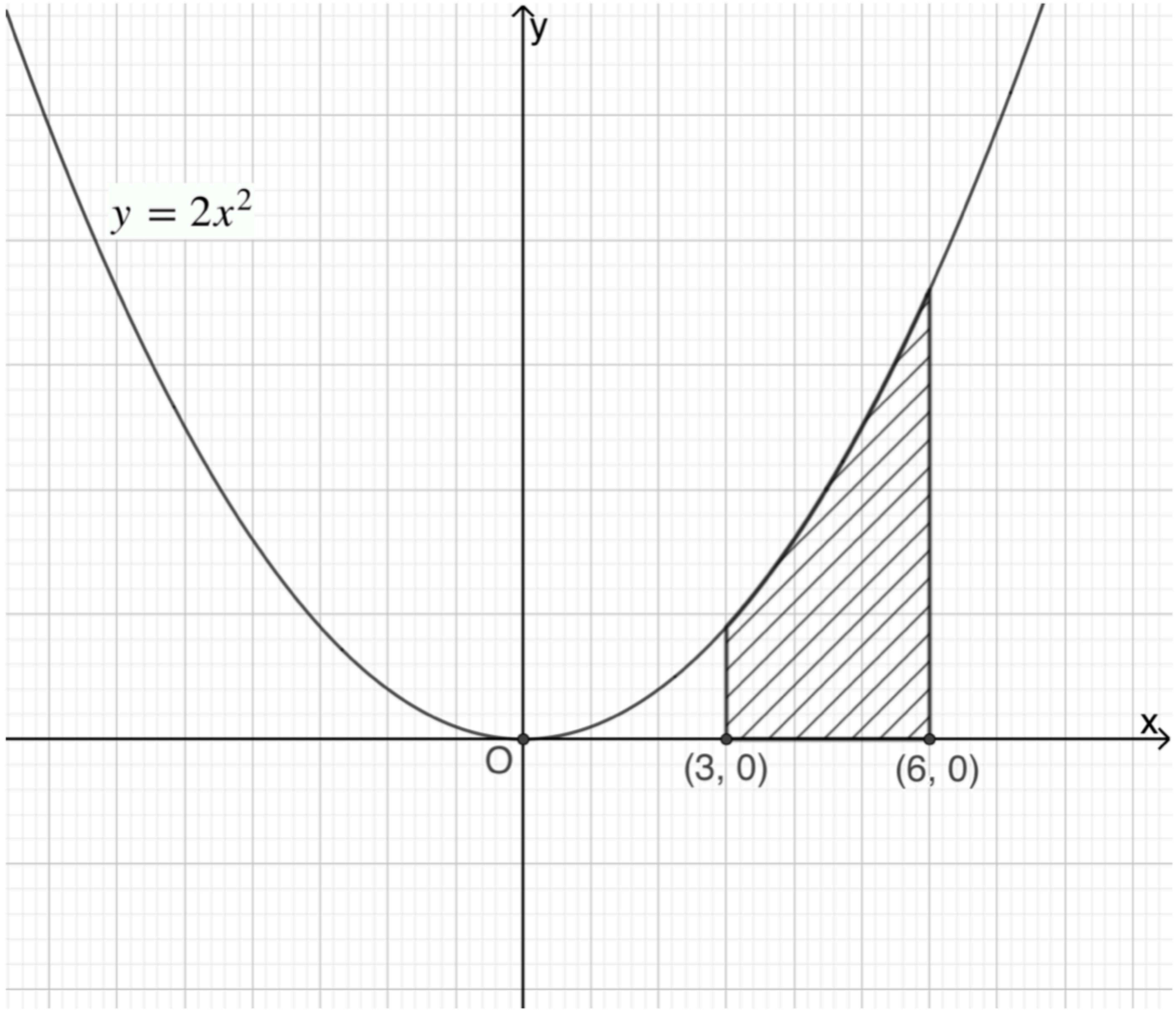
Area of region
$$= \int_a^b x\,dy$$

For a region bounded by the curve and the y-axis,
- If the region is to the left of y-axis, then the integral value is negative.
- If the region is to the right of y-axis, then the integral value is positive.
- The areas of both regions are positive.

Area between the curve and the x-axis

Find the area for each of the following shaded regions.

9. Find the area of the shaded region.

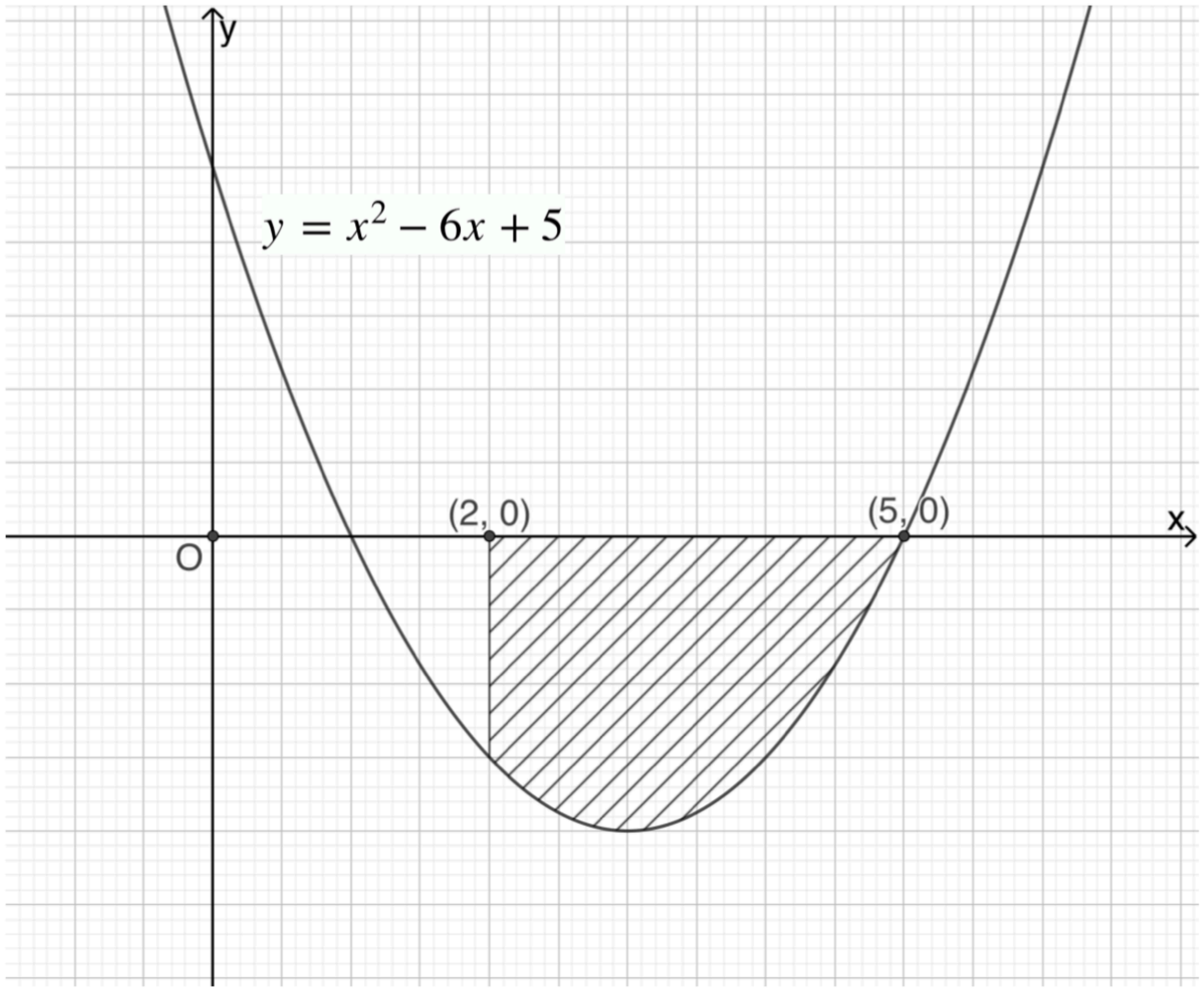


$$Area = \int_3^6 y \ dx$$
$$= \int_3^6 2x^2 \ dx$$
$$= 2\int_3^6 x^2 \ dx$$
$$= 2\left[\frac{x^3}{3}\right]_3^6$$
$$= \frac{2}{3}\left(6^3 - 3^3\right)$$
$$= 126$$

Area between the curve and the x-axis

Find the area for each of the following shaded regions.

10. Find the area of the shaded region.



Area

$$= \int_2^5 y \, dx$$

$$= \int_2^5 \left(x^2 - 6x + 5\right) dx$$

$$= \left[\frac{x^3}{3} - \frac{6x^2}{2} + 5x \right]_2^5$$

$$= \left(\frac{5^3}{3} - 3(5)^2 + 5(5) \right) - \left(\frac{2^3}{3} - 3(2)^2 + 5(2) \right)$$

$$= -\frac{25}{3} - \frac{2}{3}$$

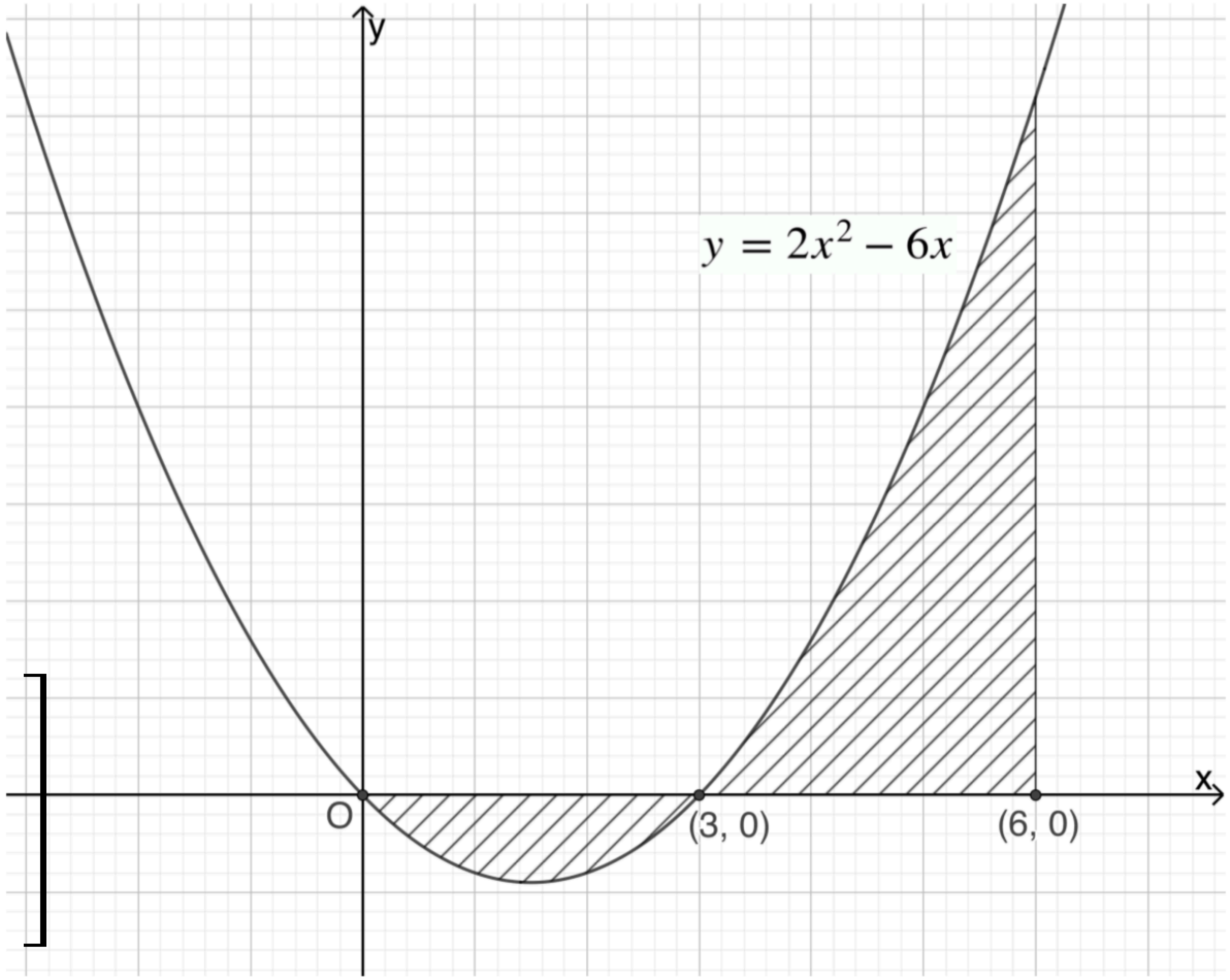
$$= \left| -9 \right|$$

$$= 9$$

Area between the curve and the x-axis

Find the area for each of the following shaded regions.

11. Find the area of the shaded regions.



Area

$$= \int_0^3 y\,dx + \int_3^6 y\,dx$$

$$= \left| \int_0^3 \left(2x^2 - 6x\right)\,dx \right| + \int_3^6 \left(2x^2 - 6x\right)\,dx$$

$$= \left| \left[\frac{2x^3}{3} - \frac{6x^2}{2} \right]_0^3 \right| + \left[\frac{2x^3}{3} - \frac{6x^2}{2} \right]_3^6$$

$$= \left| \left(\frac{2(3)^3}{3} - 3(3)^2 \right) - 0 \right| - \left[\left(\frac{2(6)^3}{3} - 3(6)^2 \right) - \left(\frac{2(3)^3}{3} - 3(3)^2 \right) \right]$$

$$= \left| 18 - 27 \right| + [\,36 - (-9)]$$

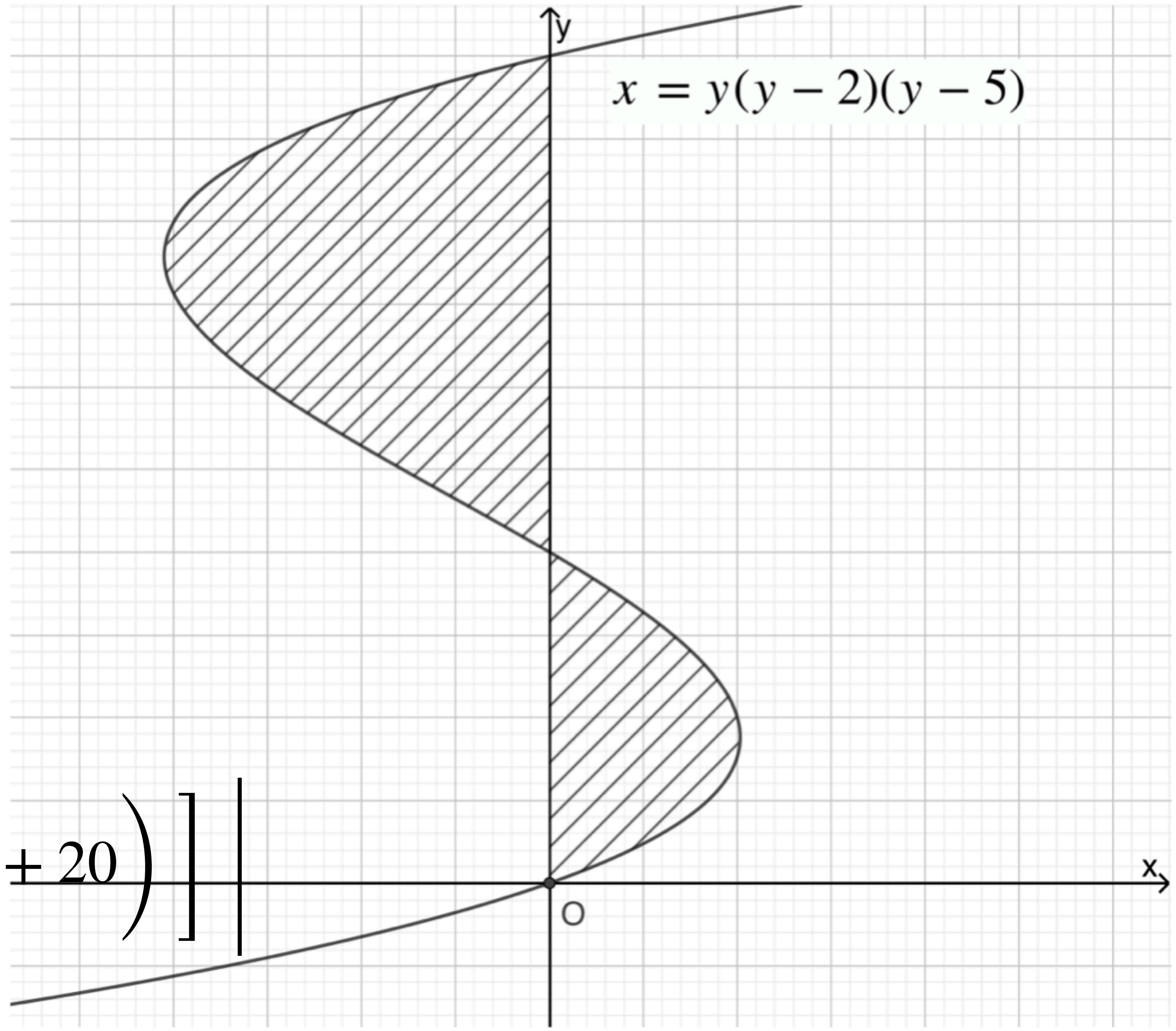
$$= \left| -9 \right| + 45$$

$$= 54$$

Area between the curve and the y-axis

Find the area for each of the following shaded regions.

12. Find the area of the shaded regions.



$$Area = \int_0^2 x\,dy + \left| \int_2^5 x\,dy \right|$$

$$x = y(y-2)(y-5)$$
$$= (y^2-2y)(y-5)$$
$$= y^3-5y^2-2y^2+10y$$
$$= y^3-7y^2+10y$$

$$= \int_0^2 (y^3-7y^2+10y)\,dy + \left| \int_2^5 (y^3-7y^2+10y)\,dy \right|$$

$$= \left[\frac{y^4}{4} - \frac{7y^3}{3} + \frac{10y^2}{2} \right]_0^2 + \left| \left[\frac{y^4}{4} - \frac{7y^3}{3} + \frac{10y^2}{2} \right]_2^5 \right|$$

$$= \left[\left(\frac{16}{4} - \frac{56}{3} + 20 \right) - 0 \right] + \left| \left[\left(\frac{625}{4} - \frac{875}{3} + 125 \right) - \left(\frac{16}{4} - \frac{56}{3} + 20 \right) \right] \right|$$

$$= \frac{16}{3} + \left| -\frac{63}{4} \right|$$

$$= \frac{253}{12}$$

$At\ y - axis\ (x = 0)$

$$y(y-2)(y-5) = 0$$
$$y = 0,\ y = 2,\ y = 5$$

Area between the curve and a straight line.

Find the area for each of the following shaded regions.

Recommended Method

$$= Area\ under\ curve - area\ of\ triangle$$

$$= \int_{-1}^{2} y\,dx - \left(\frac{1}{2} \times b \times h\right)$$

$$= \int_{-1}^{2} (4 - x^2)\,dx - \left(\frac{1}{2} \times 3 \times 3\right)$$

$$= \left[4x - \frac{x^3}{3}\right]_{-1}^{2} - \frac{9}{2}$$

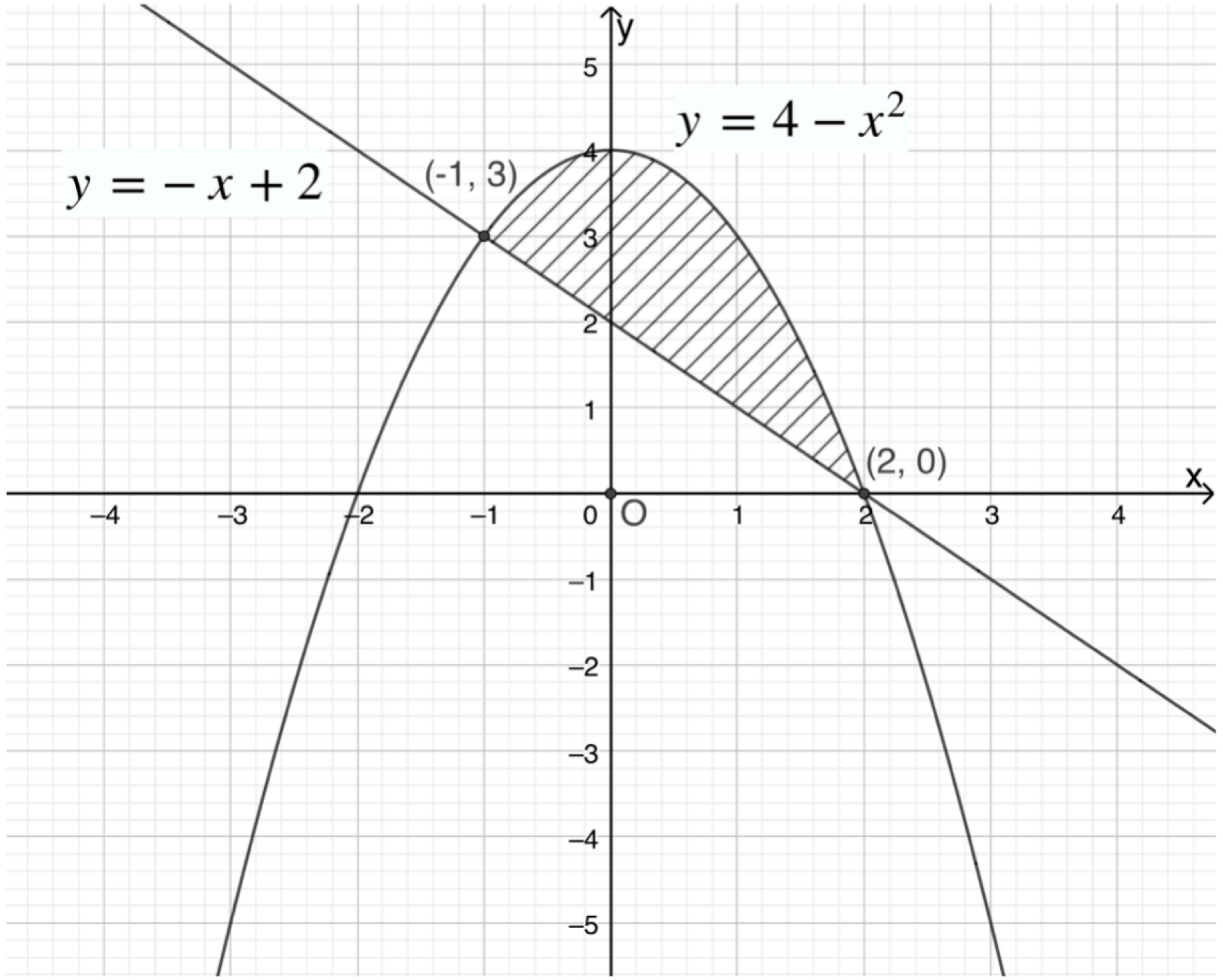
$$= \left[\left(8 - \frac{8}{3}\right) - \left(-4 + \frac{1}{3}\right)\right] - \frac{9}{2}$$

$$= \left[\left(\frac{16}{3}\right) - \left(-\frac{11}{3}\right)\right] - \frac{9}{2}$$

$$= \frac{27}{3} - \frac{9}{2}$$

$$= \frac{9}{2}$$

13. Find the area of the shaded region.



Area between the curve and a straight line.

Find the area for each of the following shaded regions.

Alternative Method

$$= \int_{-1}^{2} \left(4 - x^2 - (-x + 2)\right) dx$$

$$= \int_{-1}^{2} \left(4 - x^2 + x - 2\right) dx$$

$$= \int_{-1}^{2} \left(-x^2 + x + 2\right) dx$$

$$= \left[- \frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$$

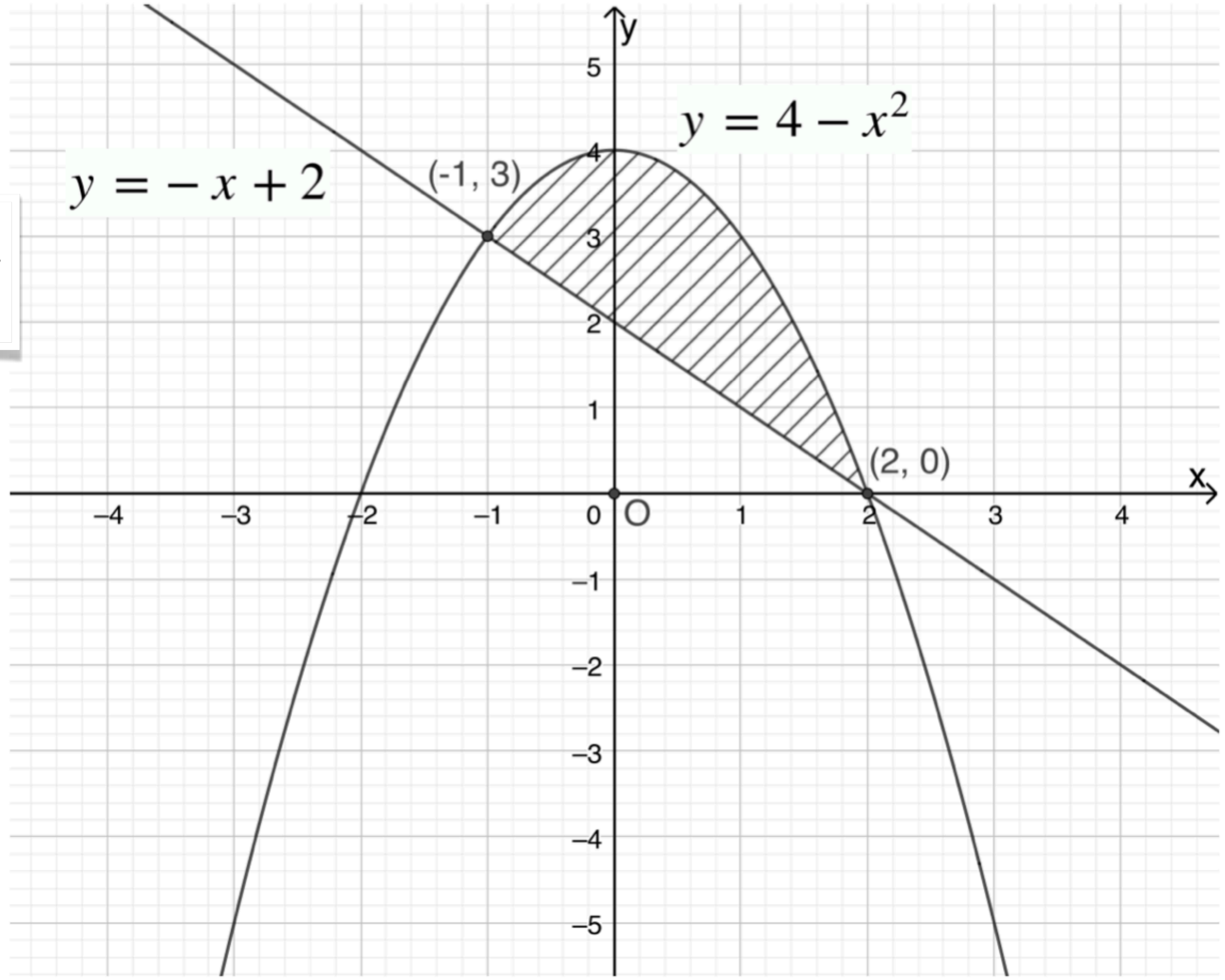
$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(-\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= \left[\frac{10}{3} - \left(-\frac{7}{6}\right)\right]$$

$$= \frac{9}{2}$$

13. Find the area of the shaded region.

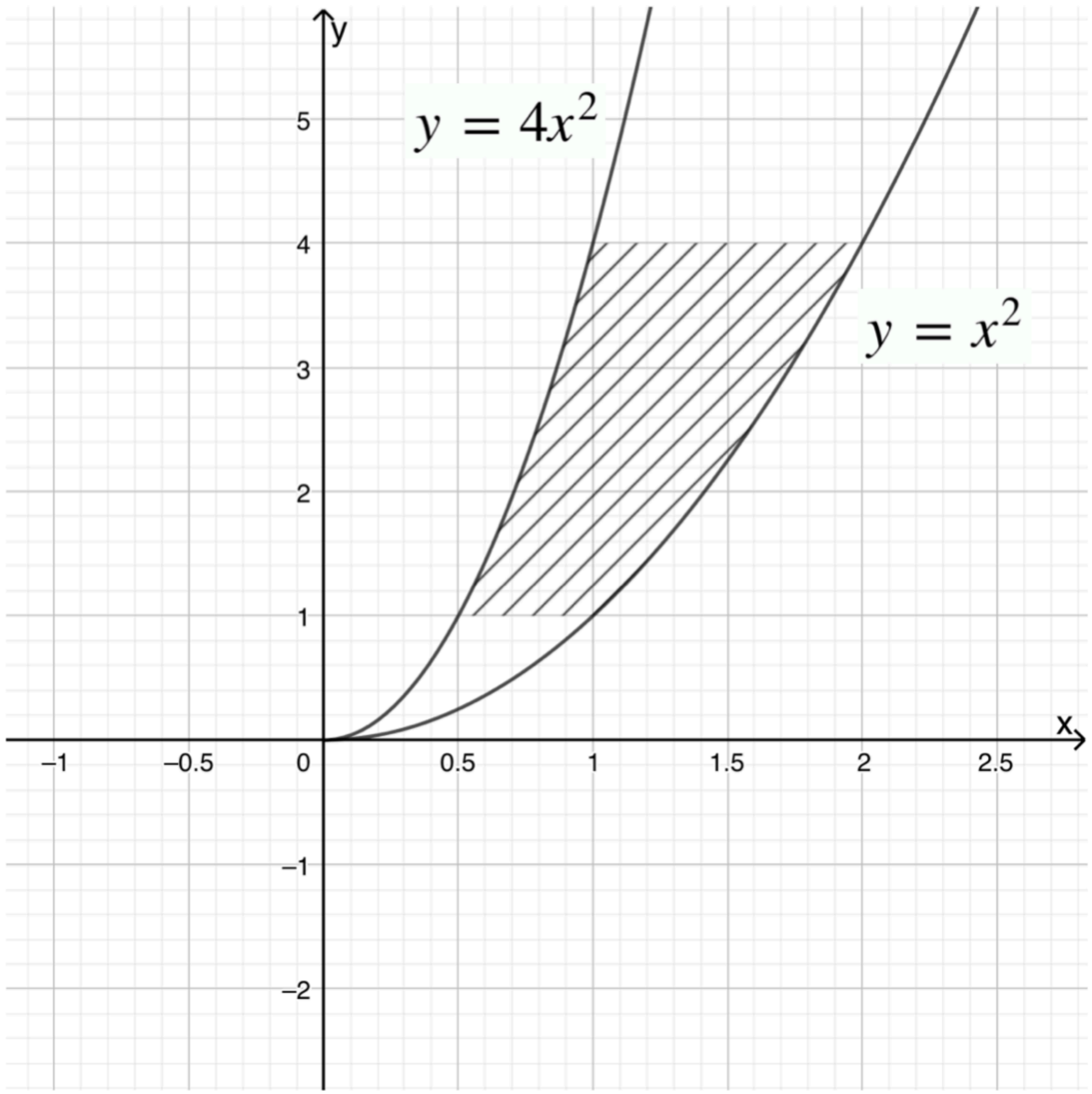
$$Area = \int_{x_1}^{x_2} Above_{function} - Lower_{function}\, dx$$



Area between the curve and a straight line.

Find the area for each of the following shaded regions.

14. Find the area of the shaded region.



$$y = 4x^2$$
$$\frac{y}{4} = x^2$$
$$\sqrt{\frac{y}{4}} = \sqrt{x^2}$$
$$x = \sqrt{\frac{y}{4}}$$
$$x = \frac{\sqrt{y}}{2}$$

$$y = x^2$$
$$\sqrt{y} = \sqrt{x^2}$$
$$x = \sqrt{y}$$

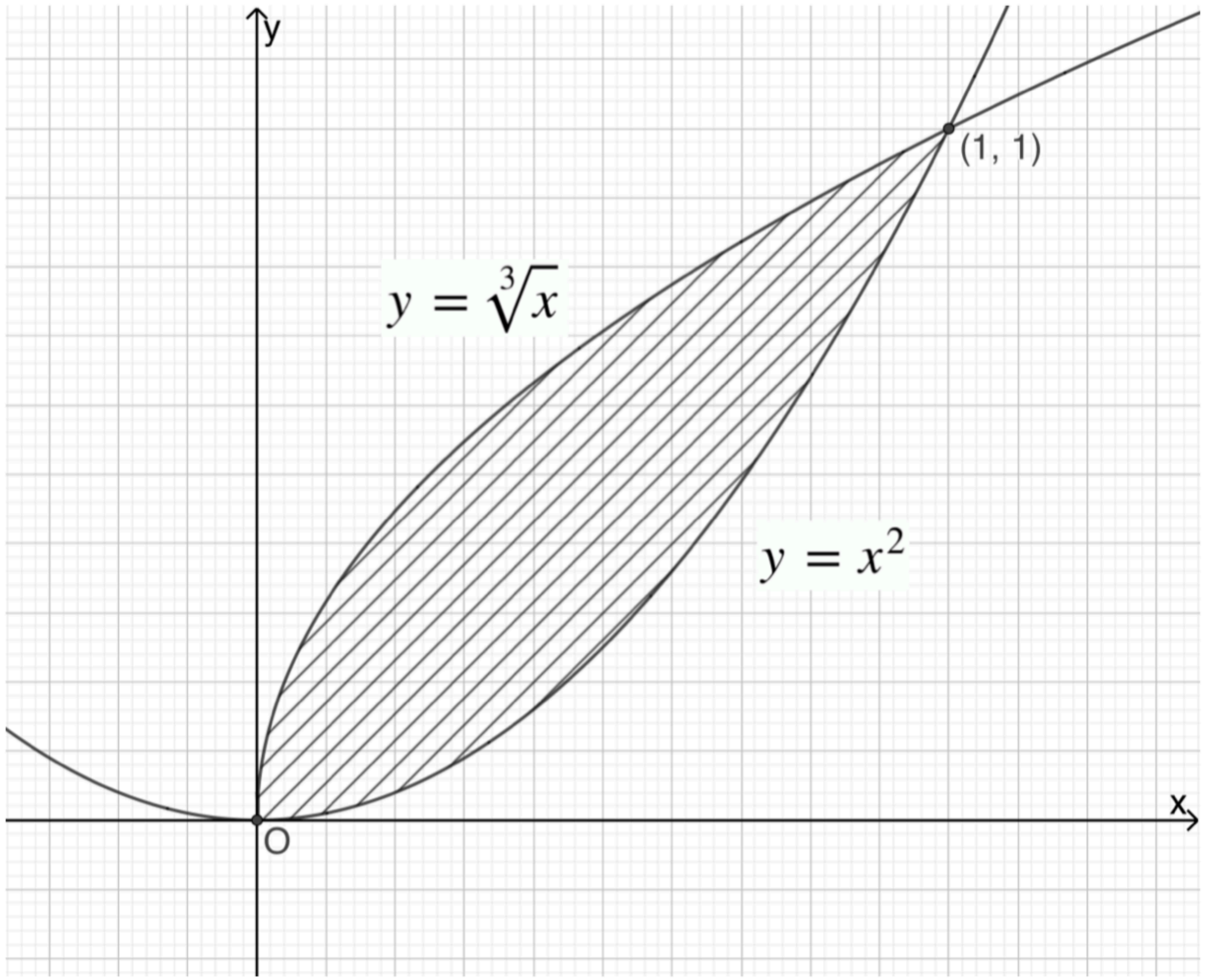
$$= \int_1^4 \sqrt{y}\ dy - \int_1^4 \frac{\sqrt{y}}{2}\, dy$$
$$= \int_1^4 y^{\frac{1}{2}}\ dy - \int_1^4 \frac{y^{\frac{1}{2}}}{2}\, dy$$
$$= \int_1^4 \left(y^{\frac{1}{2}} - \frac{y^{\frac{1}{2}}}{2} \right) dy$$
$$= \frac{1}{2}\int_1^4 y^{\frac{1}{2}}\, dy$$
$$= \frac{1}{2}\left[\frac{2y^{\frac{3}{2}}}{3} \right]_1^4$$
$$= \frac{1}{3}\left[y^{\frac{3}{2}} \right]_1^4$$
$$= \frac{1}{3}(8 - 1)$$
$$= \frac{7}{3}$$

Area between two curves.

Find the area for each of the following shaded regions.

Integrate with respect to x − axis

15. Find the area of the shaded region.



$$= \int_0^1 \sqrt[3]{x}\ dx - \int_0^1 x^2\, dx$$

$$= \int_0^1 x^{\frac{1}{3}}\ dx - \int_0^1 x^2\, dx$$

$$= \int_0^1 \left(x^{\frac{1}{3}} - x^2\right) dx$$

$$= \left[\frac{3x^{\frac{4}{3}}}{4} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{3}{4}(1)^{\frac{4}{3}} - \frac{(1)^3}{3}\right) - 0$$

$$= \frac{3}{4} - \frac{1}{3}$$

$$= \frac{5}{12}$$

Area between two curves.

Find the area for each of the following shaded regions.

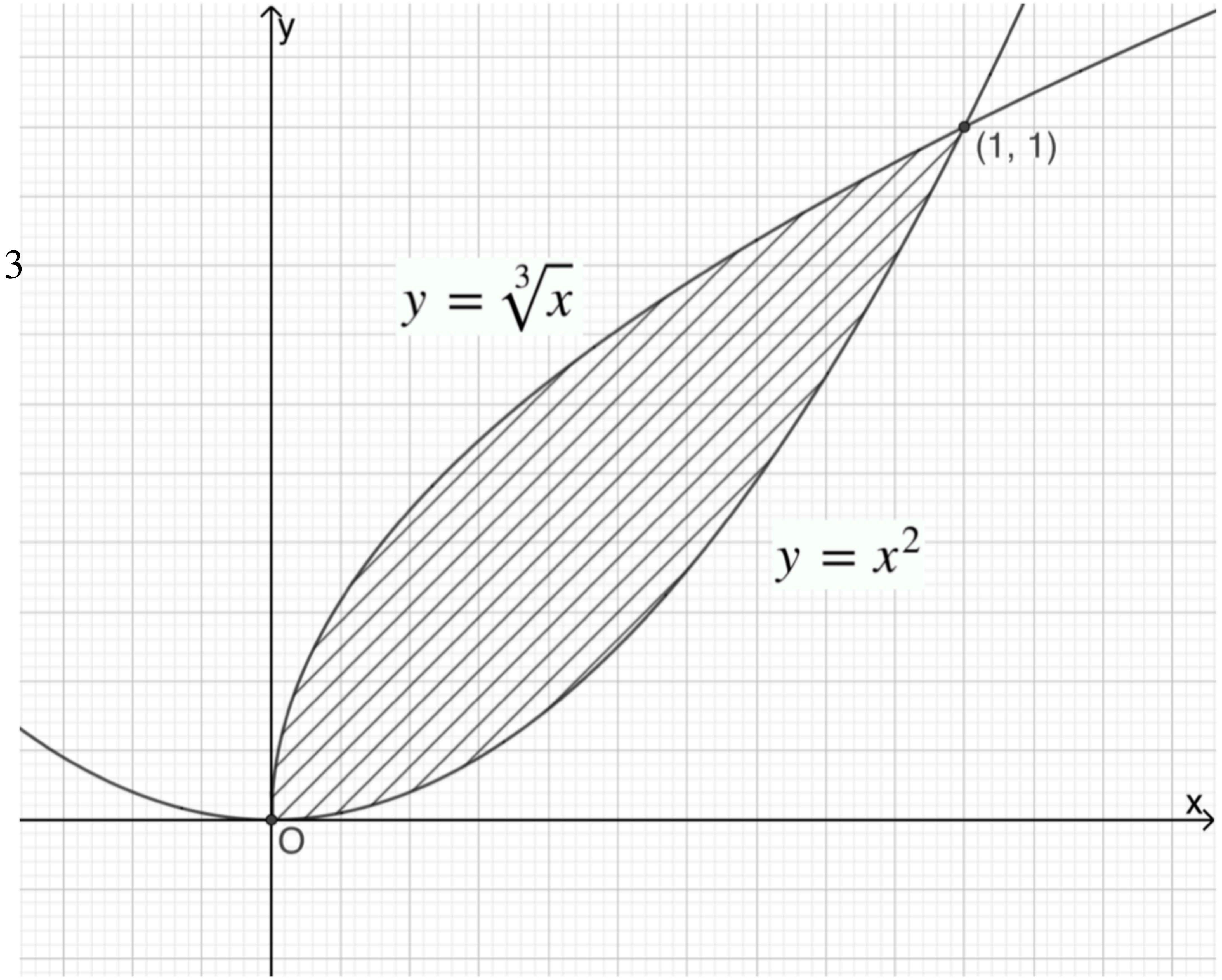
Integrate with respect to y − axis

15. Find the area of the shaded region.

$y = \sqrt[3]{x}$

$y^3 = \left(\sqrt[3]{x}\right)^3$

$x = y^3$



$y = x^2$

$\sqrt{y} = \sqrt{x^2}$

$x = \sqrt{y}$

$$= \int_0^1 \sqrt{y}\, dy - \int_0^1 y^3\, dy$$

$$= \int_0^1 y^{\frac{1}{2}}\, dy - \int_0^1 y^3\, dy$$

$$= \int_0^1 \left(y^{\frac{1}{2}} - y^3\right) dy$$

$$= \left[\frac{2y^{\frac{3}{2}}}{3} - \frac{y^4}{4}\right]_0^1$$

$$= \left(\frac{2}{3}(1)^{\frac{3}{2}} - \frac{(1)^4}{4}\right) - 0$$

$$= \frac{2}{3} - \frac{1}{4}$$

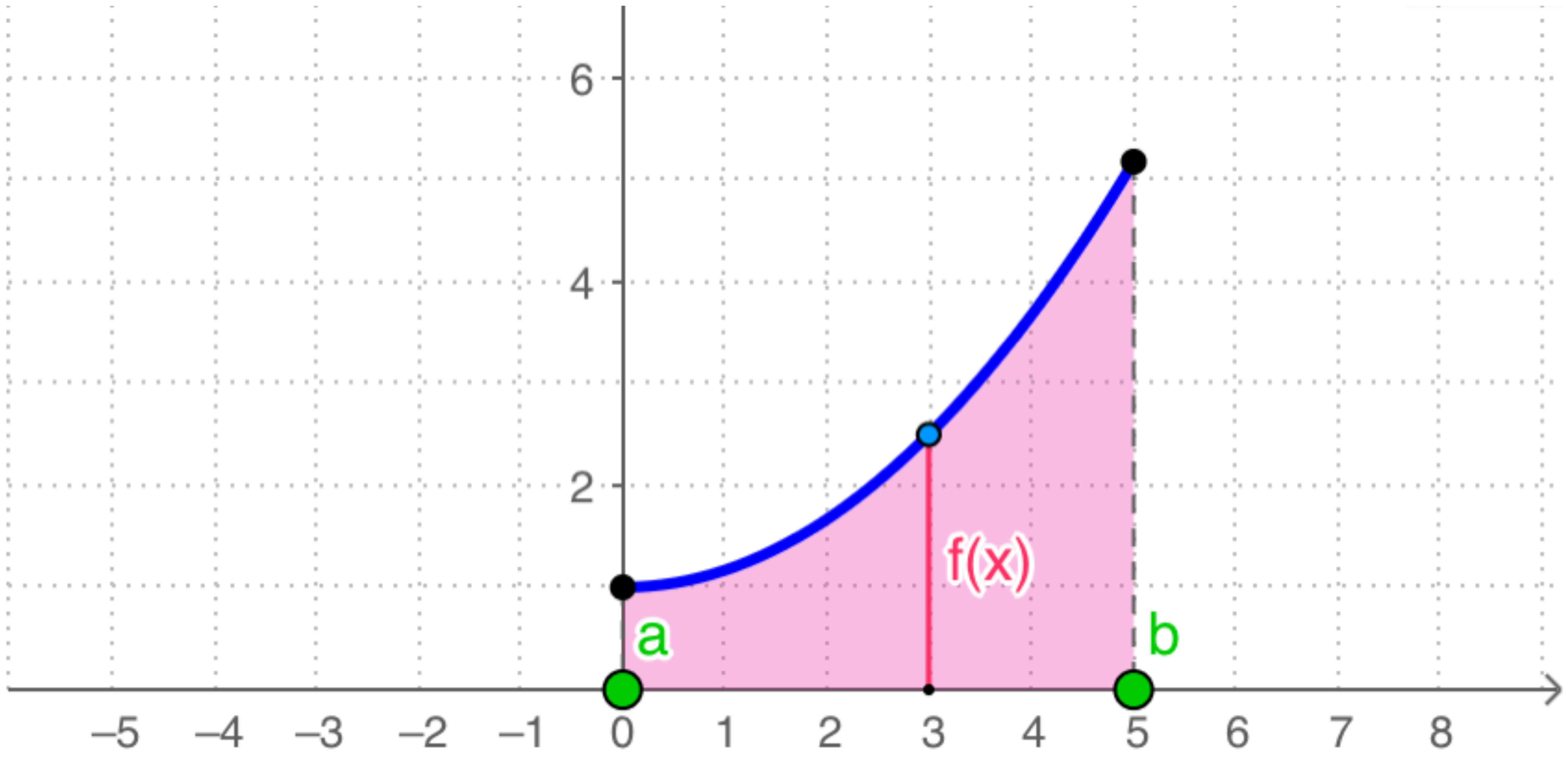
$$= \frac{5}{12}$$

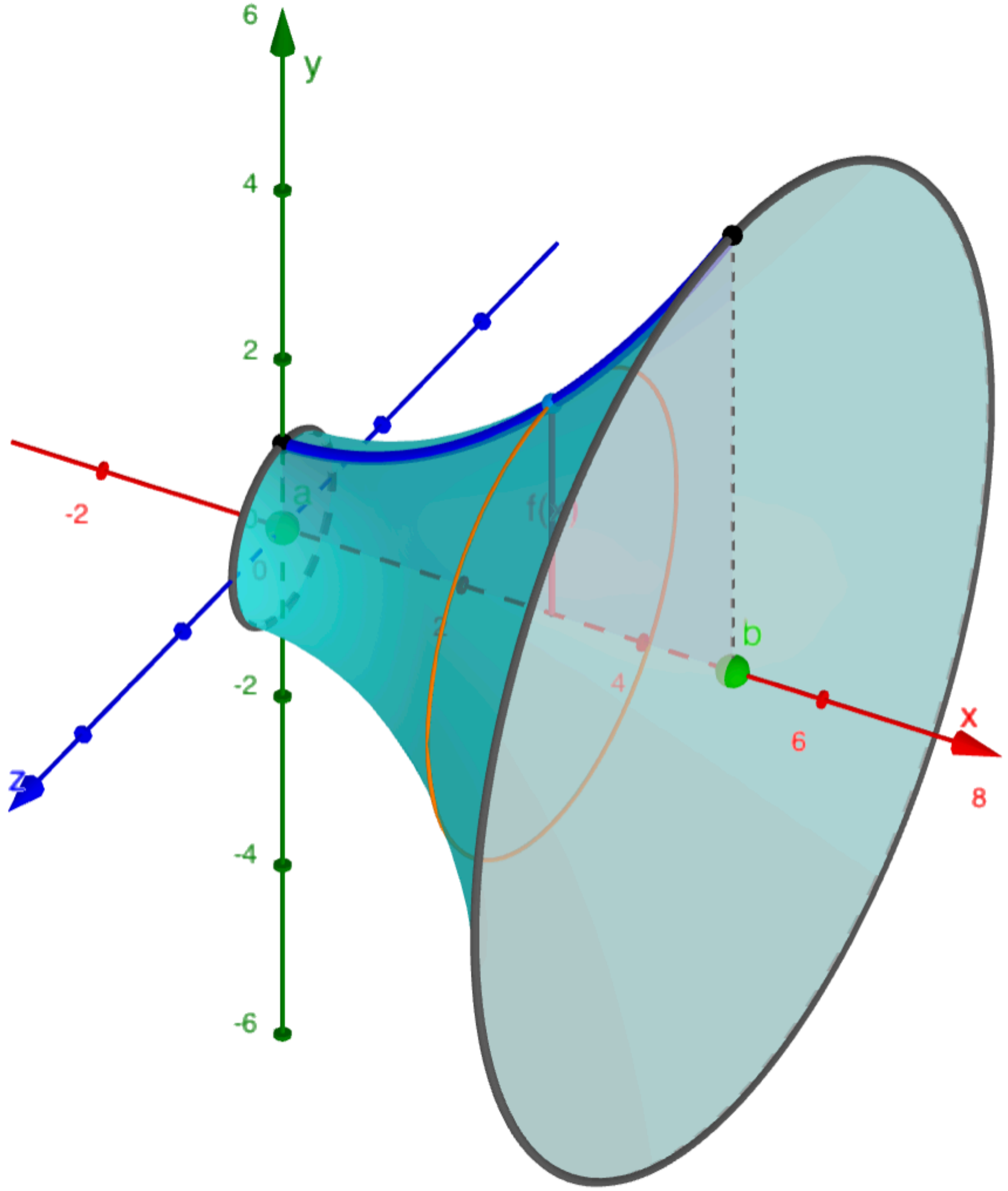
Generated Volume

The generated volume of a region revolved at the x-axis or the y-axis

The generated volume V when a region bounded by the curve y = f(x) enclosed by x = a and x = b is revolved through 360° about the x-axis is given by:

$$V = \int_a^b \pi y^2 dx$$

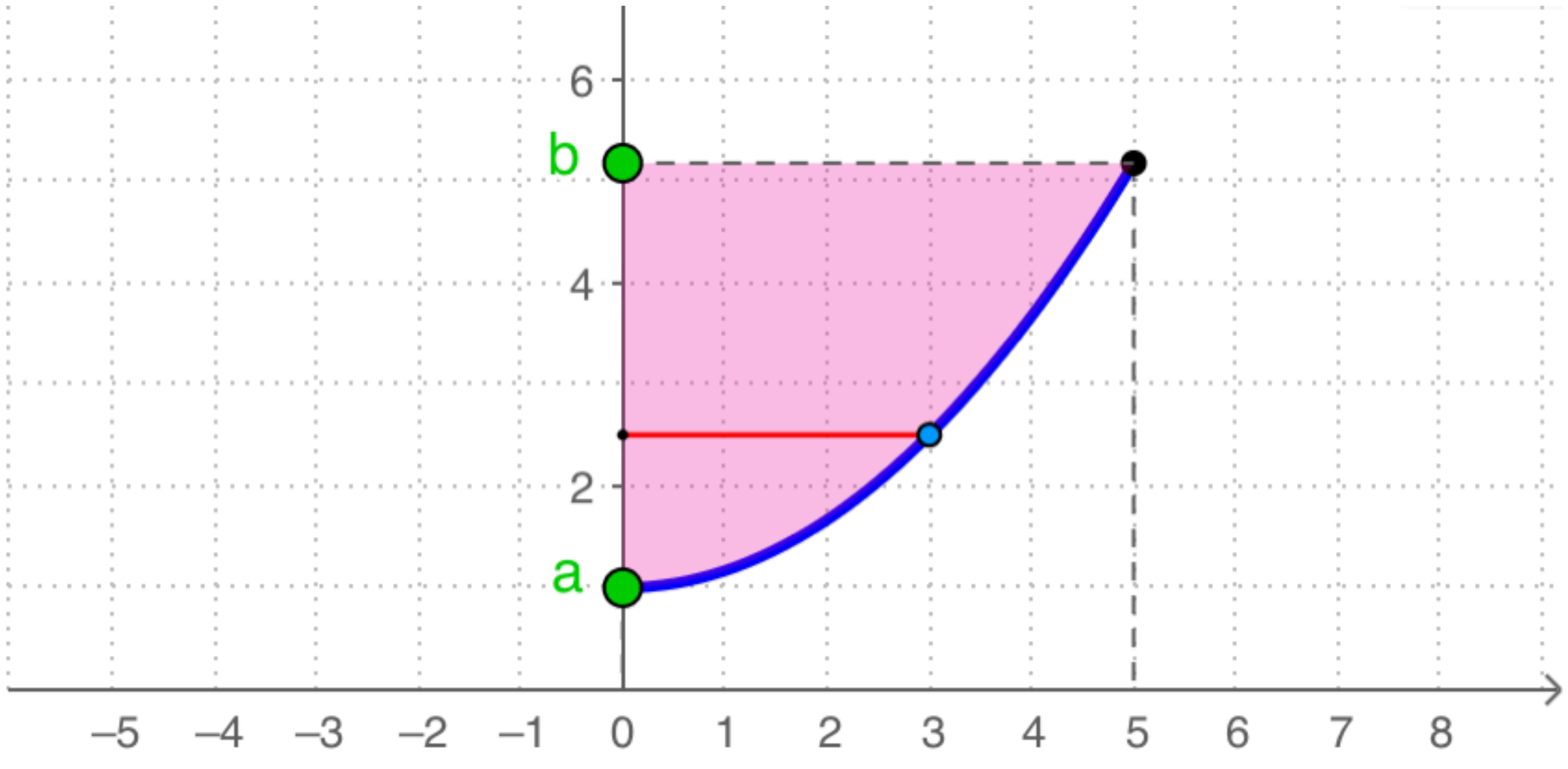


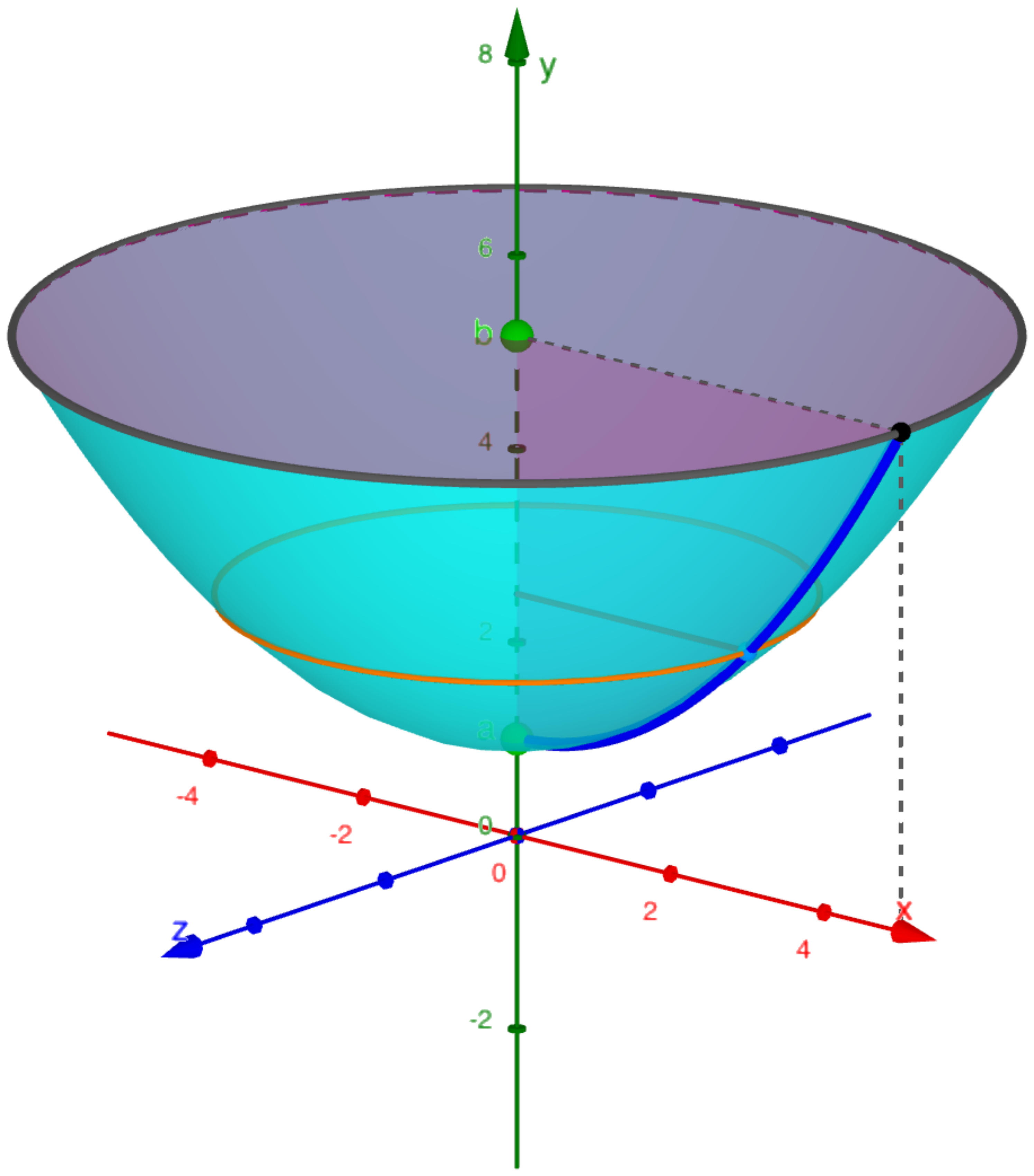


The generated volume of a region revolved at the x-axis or the y-axis

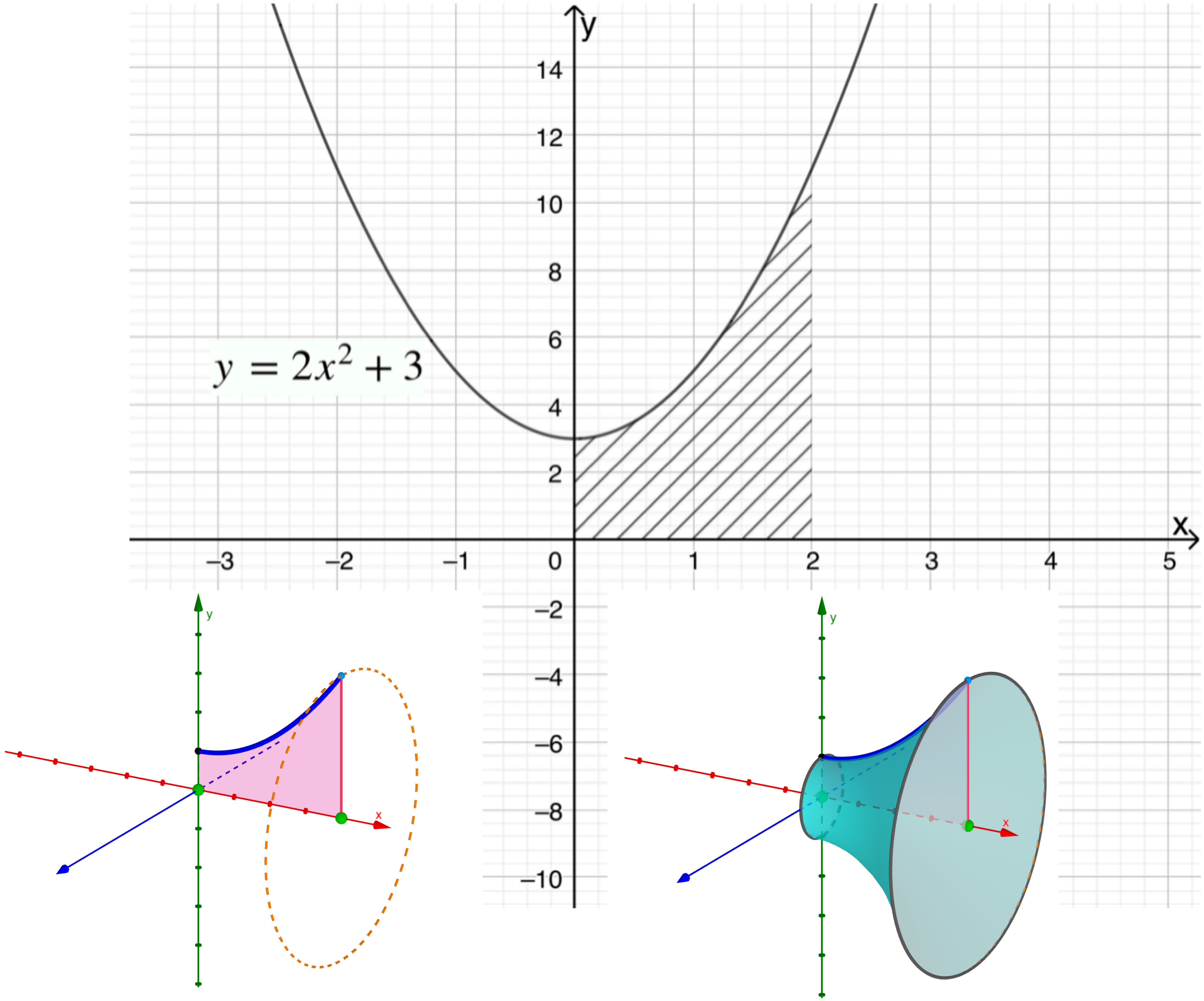
The generated volume V when a region bounded by the curve x = g(y) enclosed by y = a and y = b is revolved through 360° about the y-axis is given by:

$$V = \int_a^b \pi x^2 dy$$





16. Find the generated volume, in terms of π, when the shaded region in the diagram is rotated through 360° about x - axis.



Volume

$$= \int_0^2 \pi y^2 \; dx$$

$$= \pi \int_0^2 \left(2x^2 + 3\right)^2 \; dx$$

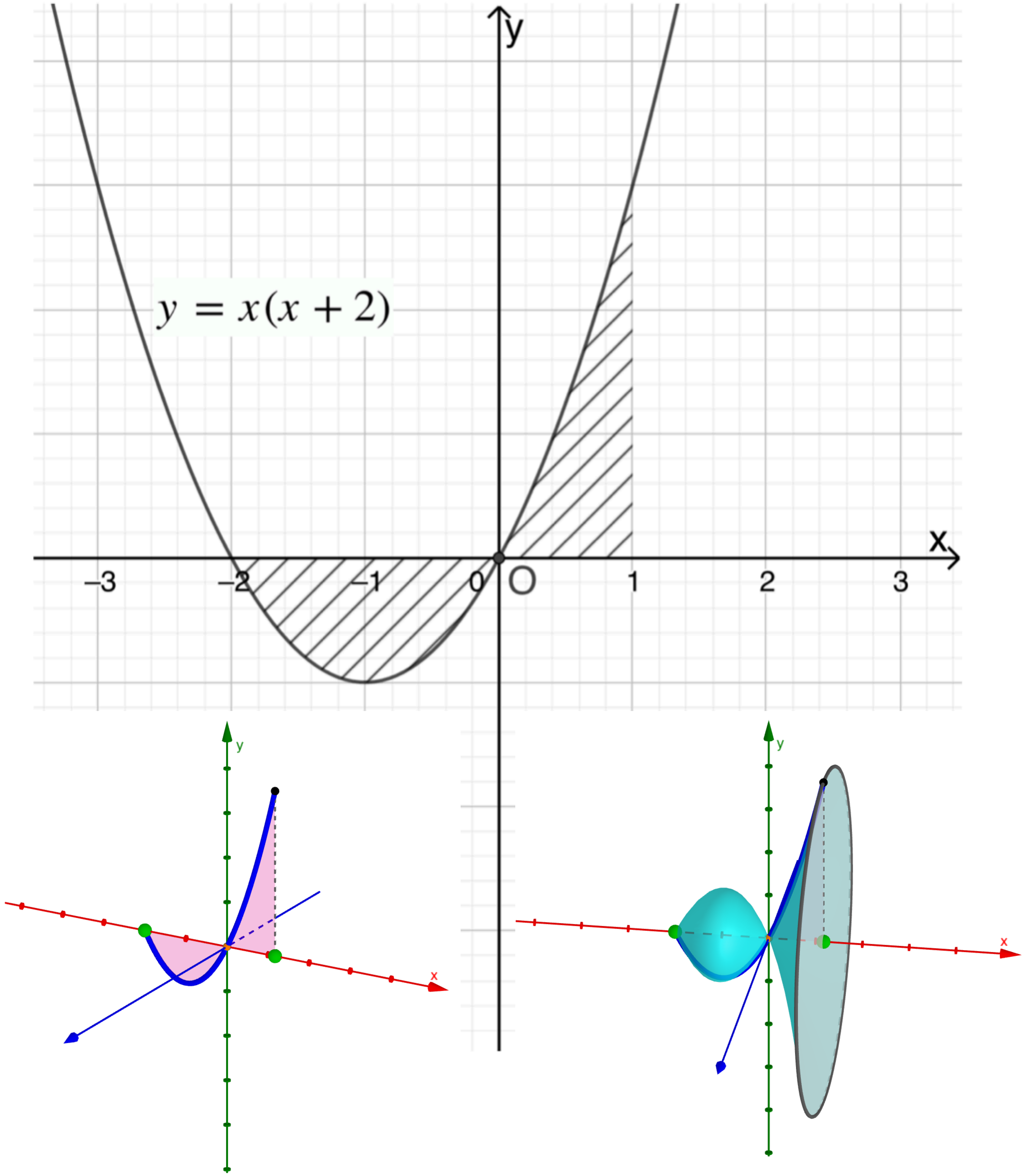
$$= \pi \int_0^2 \left(4x^4 + 12x^2 + 9\right) dx$$

$$= \pi \left[\frac{4x^5}{5} + \frac{12x^3}{3} + 9x \right]_0^2$$

$$= \pi \left[\left(\frac{128}{5} + 32 + 18 \right) - 0 \right]$$

$$= \frac{378}{5}\pi$$

17. Find the generated volume, in terms of π, when the shaded regions in the diagram are rotated through 360° about x - axis.



Volume

$$V = \int_{-2}^{1} \pi y^2 \, dx$$

$$y = x(x+2)$$

$$y = x^2 + 2x$$

$$y^2 = \left(x^2 + 2x\right)^2$$

$$\therefore y^2 = x^4 + 4x^3 + 4x^2$$

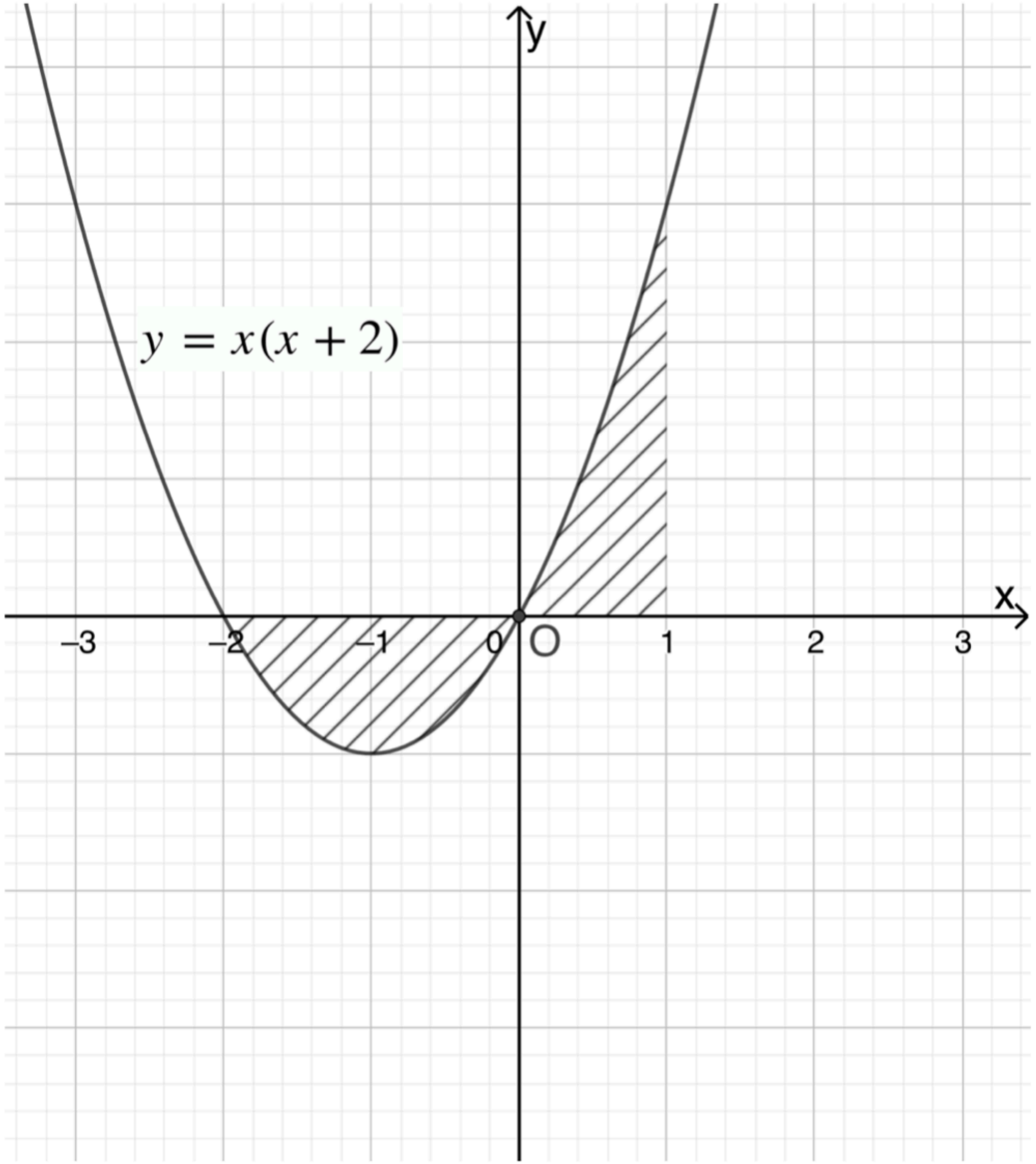
$$= \pi \int_{-2}^{1} \left(x^4 + 4x^3 + 4x^2\right) \, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{4x^4}{4} + \frac{4x^3}{3} \right]_{-2}^{1}$$

$$= \pi \left[\left(\frac{1}{5} + 1 + \frac{4}{3} \right) - \left(-\frac{32}{5} + 16 - \frac{32}{3} \right) \right]$$

$$= \pi \left[\frac{38}{15} - \left(-\frac{16}{15} \right) \right]$$

$$= \frac{18}{5}\pi$$



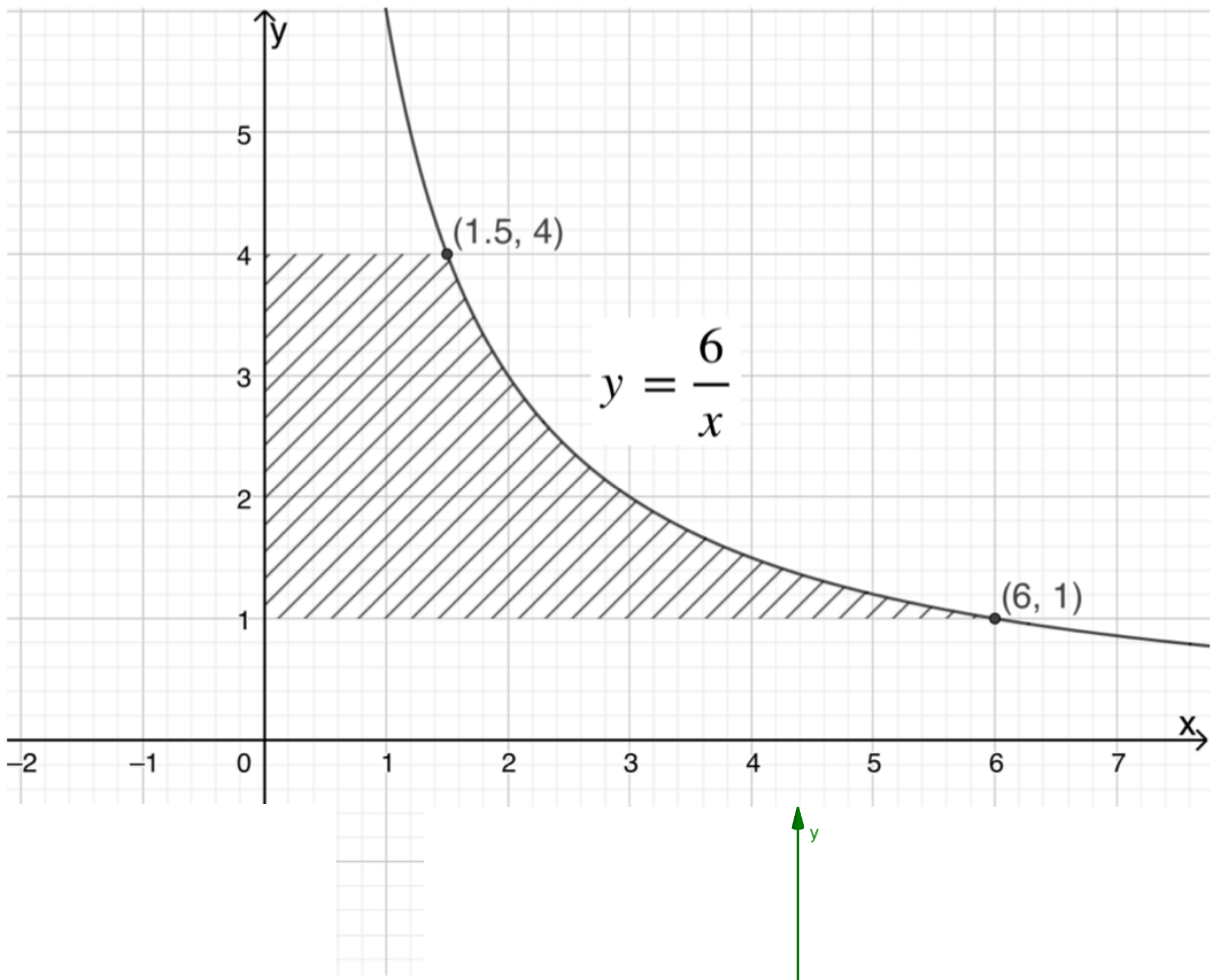
Volume:

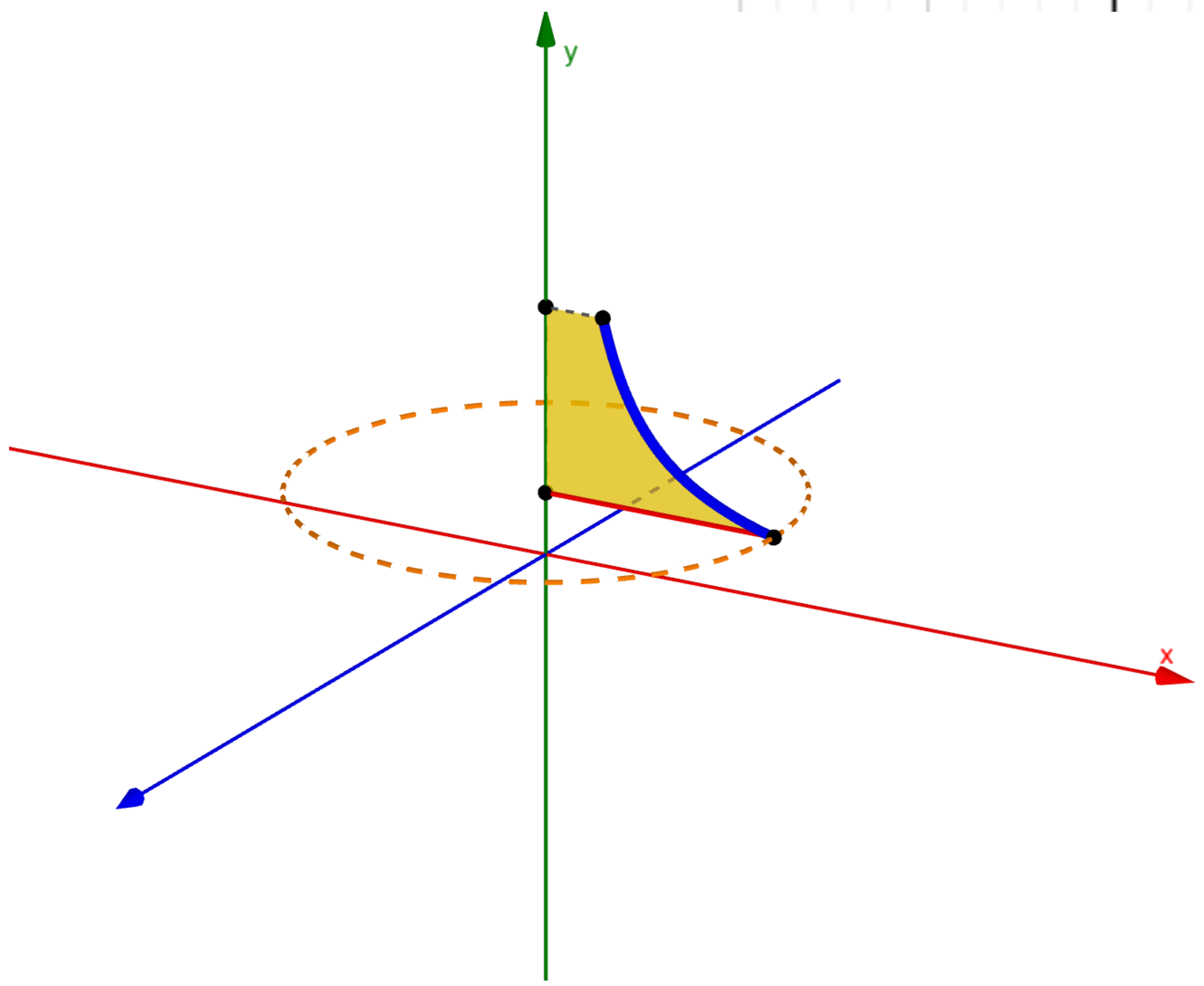
$$V = \int_{-2}^{1} \pi\, y^2 \; dx$$

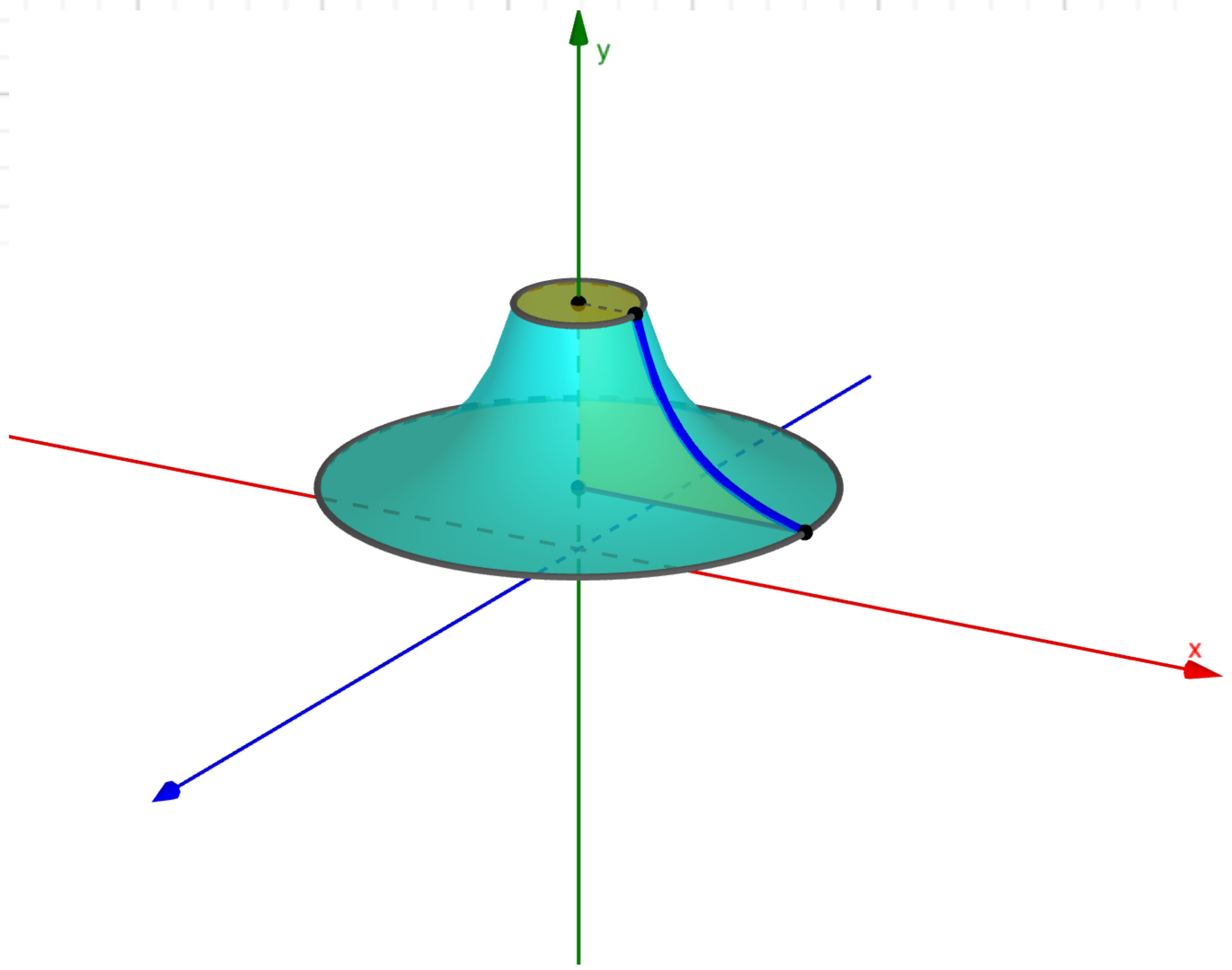
Area:

$$A = \left| \int_{-2}^{0} y \; dx \right| + \int_{0}^{1} y \; dx$$

18. Find the generated volume, in terms of π, when the shaded region in the diagram is rotated through 360° about y - axis.







$$Volume = \int_1^4 \pi\, x^2\; dy$$

$$y = \frac{6}{x};\; x = \frac{6}{y}$$

$$= \pi \int_1^4 \left(\frac{6}{y}\right)^2\; dy$$

$$= 36\pi \int_1^4 \frac{1}{y^2}\, dy$$

$$= 36\pi \int_1^4 y^{-2}\; dy$$

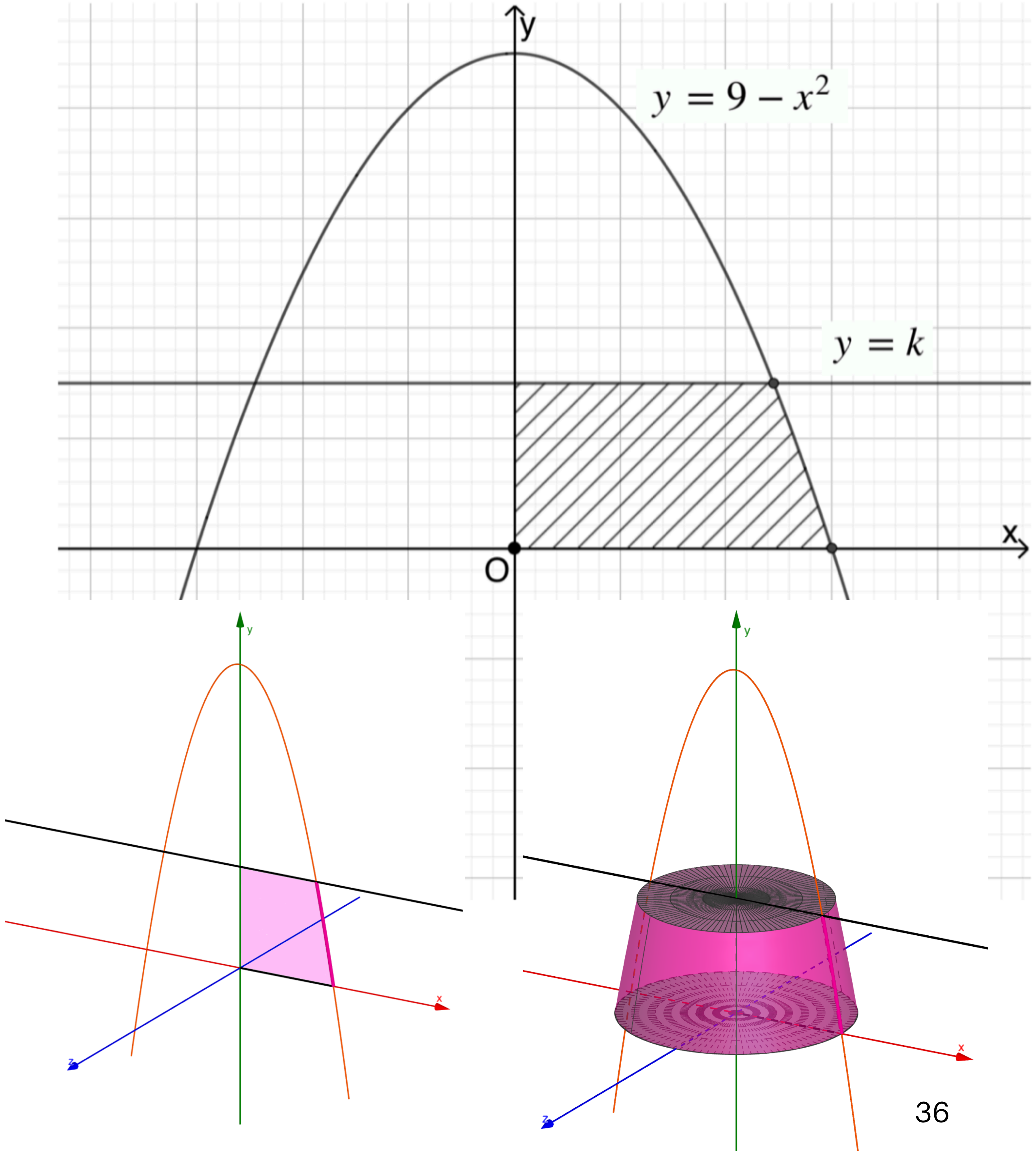
$$= 36\pi \left[\frac{y^{-1}}{-1}\right]_1^4$$

$$= -\,36\pi\left(\frac{1}{4} - 1\right)$$

$$= -\,36\pi\left(-\frac{3}{4}\right)$$

$$= 27\pi$$

19. The diagram shows a shaded region enclosed by the curve $y = 9 - x^2$, the straight line $y = k$, the y-axis and the x-axis. When the shaded region is revolved through 360° about the y-axis, the volume generated is $22\frac{1}{2}\pi$ units³. Find the value of k.



Volume

$$V = \pi \int_0^k x^2 \, dy$$

$$y = 9 - x^2$$

$$x^2 = 9 - y$$

$$\frac{45}{2}\pi = \pi \int_0^k (9 - y)\, dy$$

$$\frac{45}{2} = \left[9y - \frac{y^2}{2} \right]_0^k$$

$$\frac{45}{2} = \left(9k - \frac{k^2}{2} \right) - 0$$

$$45 = 18k - k^2$$

$$k^2 - 18k + 45 = 0$$

$$(k - 3)(k - 15) = 0$$

$$k = 3 \, , \; k = 15$$

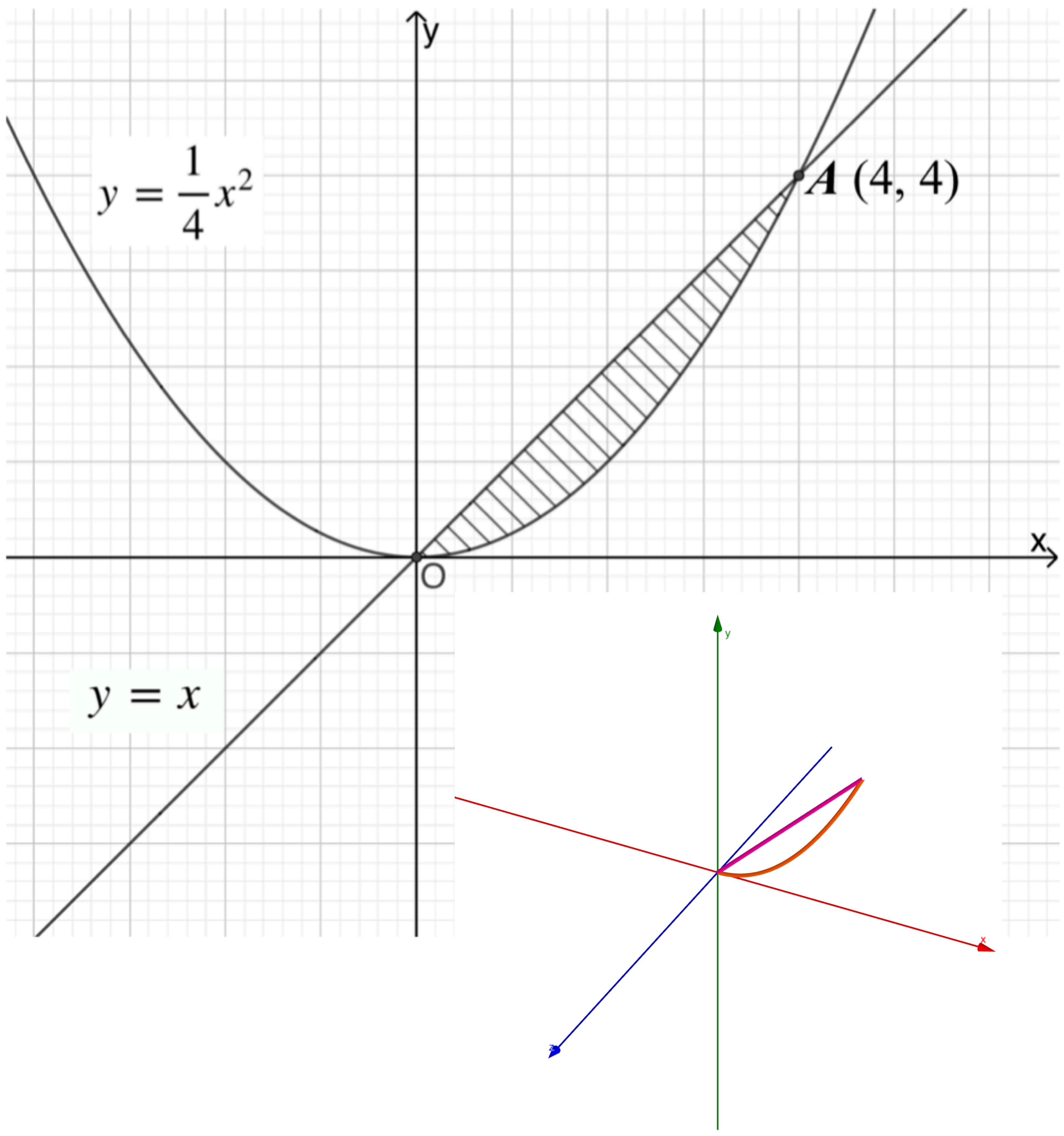
$$\therefore k = 3$$

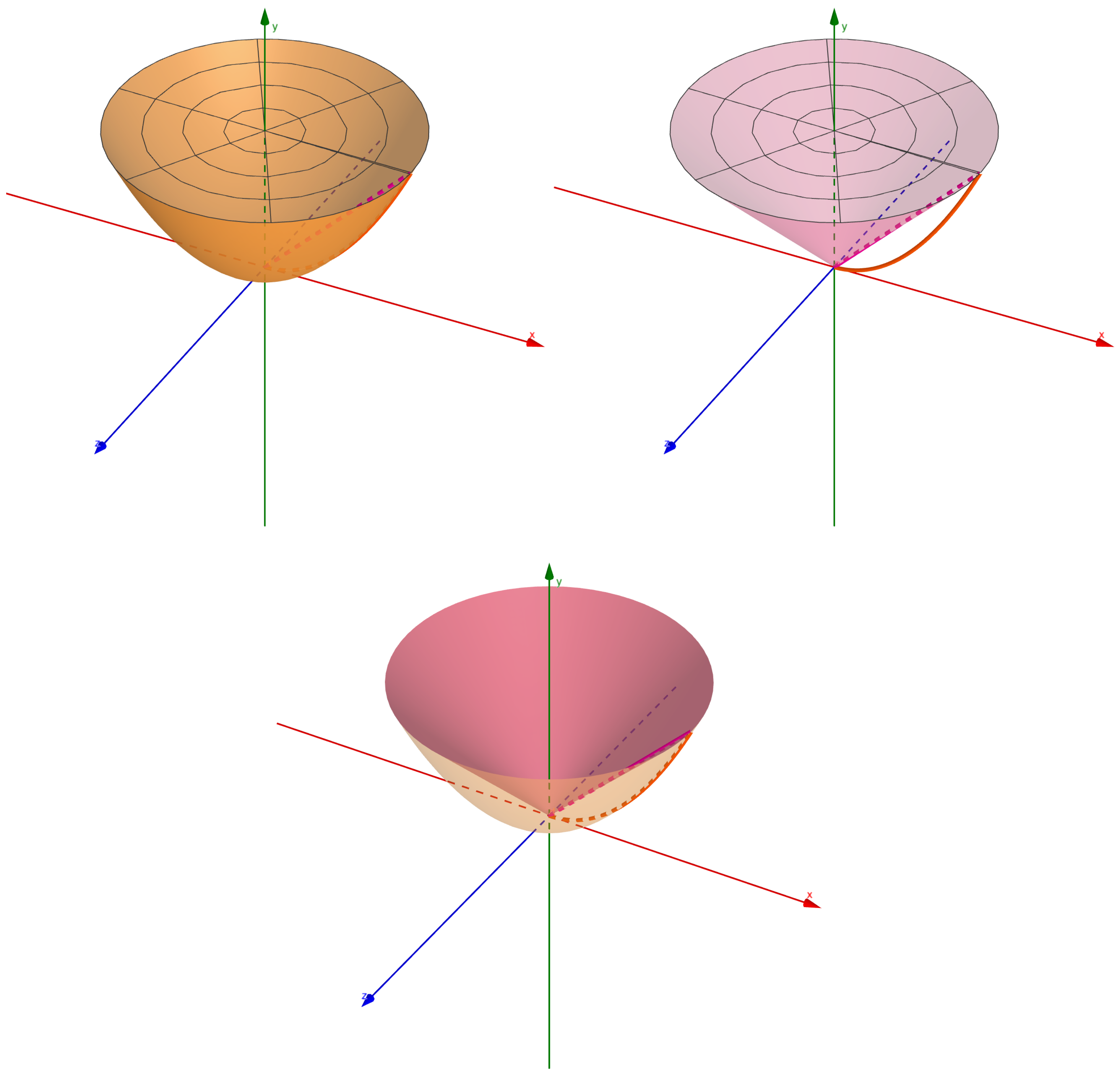
20. The diagram shows, the curve $\frac{1}{4}x^2$ intersects the straight line $y = x$ at O and $A(4, 4)$. Find

(a) the generated volume, in terms of π, when the shaded region is revolved fully about the y-axis.

(b) the generated volume, in terms of π, when the shaded region is revolved fully about the x-axis.

$$a)\ Generated\ Volume\ =\ Vol.\ by\ Curve\ -\ Vol.\ of\ Cone$$





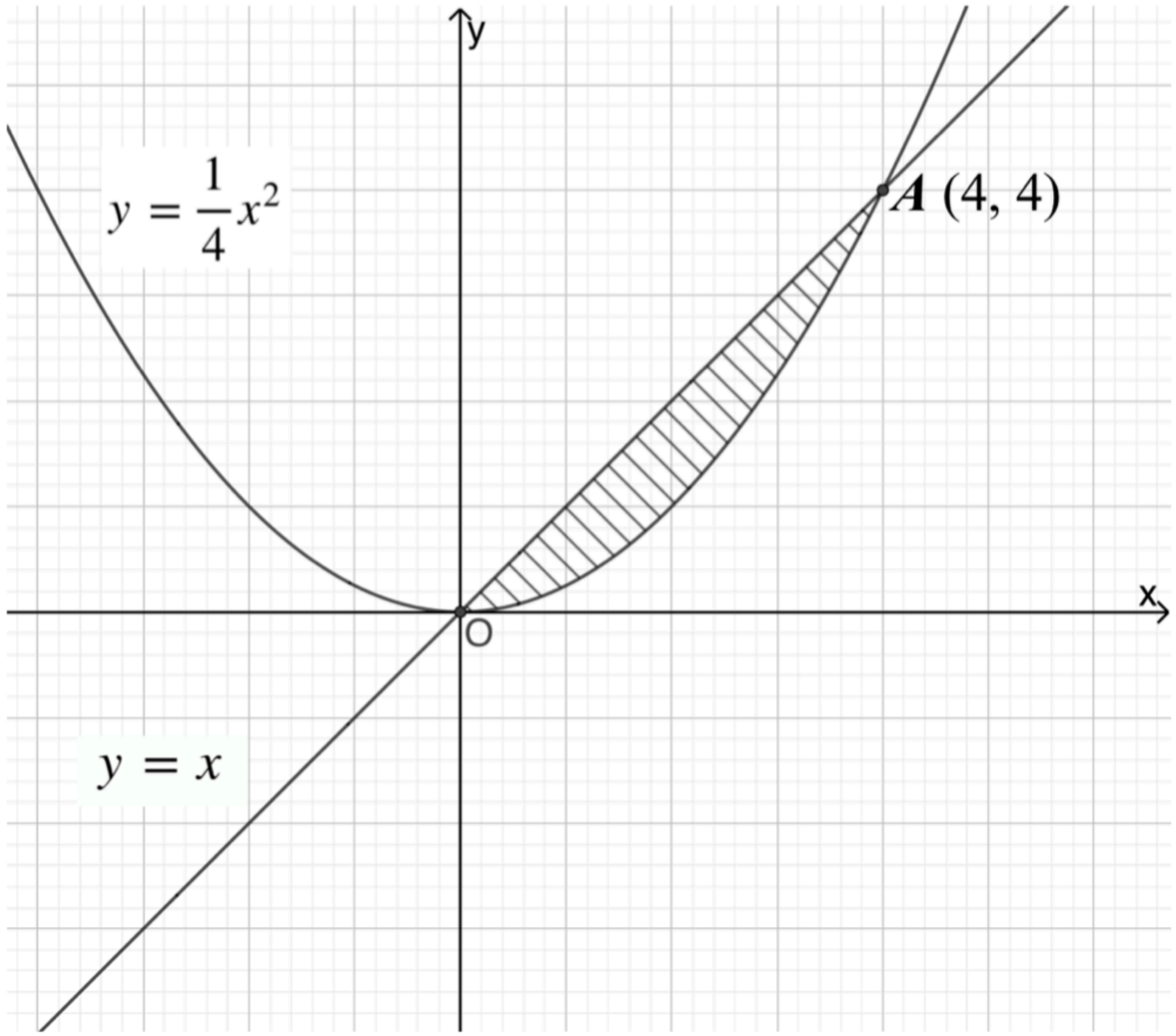
20. The diagram shows, the curve $\frac{1}{4}x^2$ intersects the straight line $y = x$ at O and $A(4, 4)$. Find

(a) the generated volume, in terms of π, when the shaded region is revolved fully about the y-axis.

(b) the generated volume, in terms of π, when the shaded region is revolved fully about the x-axis.

$$y = \frac{1}{4}x^2$$

$$x^2 = 4y$$



$$a)\ Generated\ Volume = Vol.\ by\ Curve - Vol.\ of\ Cone$$

$$= \int_0^4 \pi x^2\, dy - \frac{1}{3}\pi r^2 h$$

$$= \pi \int_0^4 x^2\, dy - \frac{1}{3}\pi(4^2)4$$

$$= \pi \int_0^4 4y\, dy - \frac{64}{3}\pi$$

$$= 4\pi \left[\frac{y^2}{2}\right]_0^4 - \frac{64}{3}\pi$$

$$= 4\pi(8 - 0) - \frac{64}{3}\pi$$

$$= 32\pi - \frac{64}{3}\pi$$

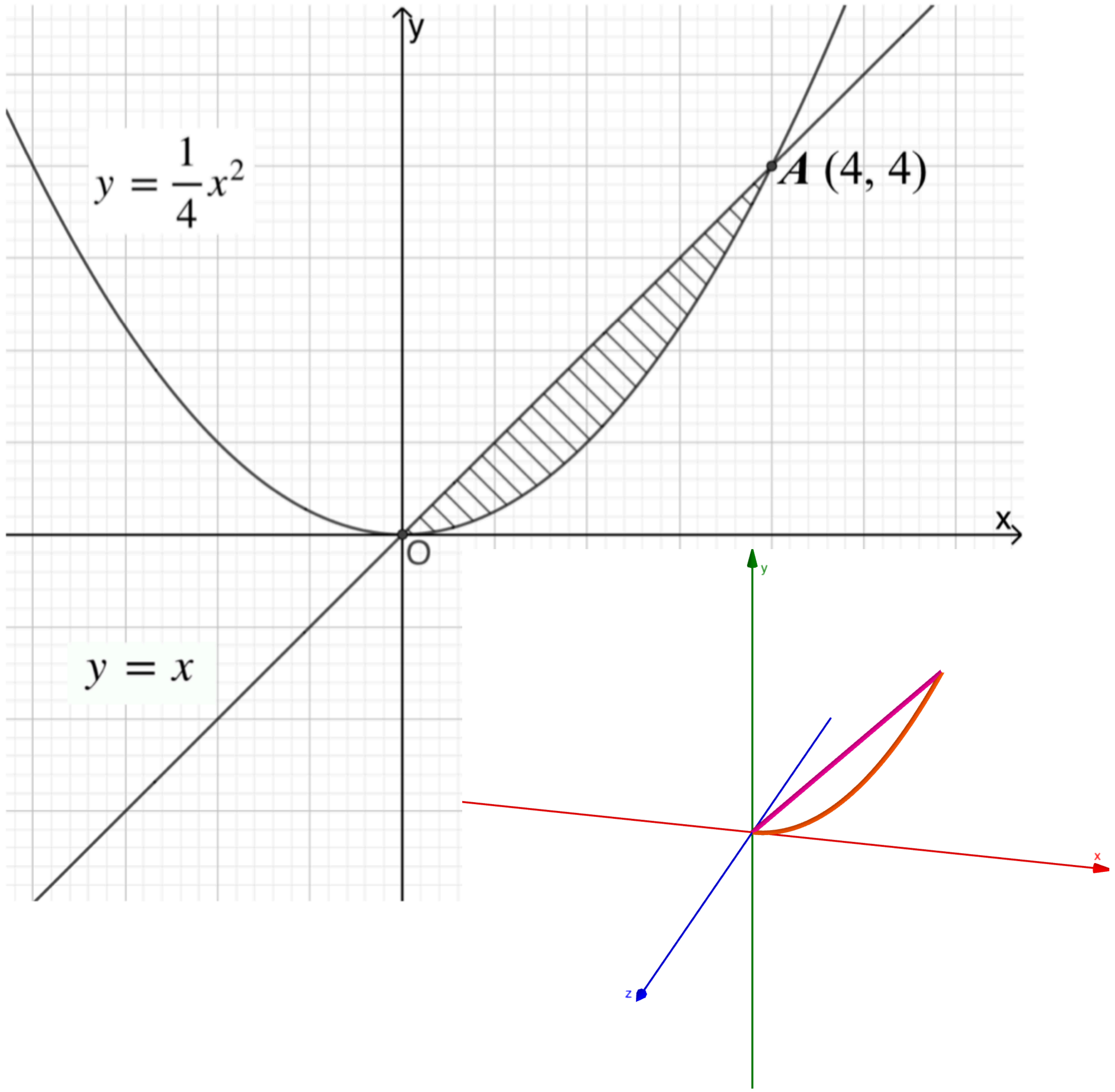
$$= \frac{32}{3}\pi$$

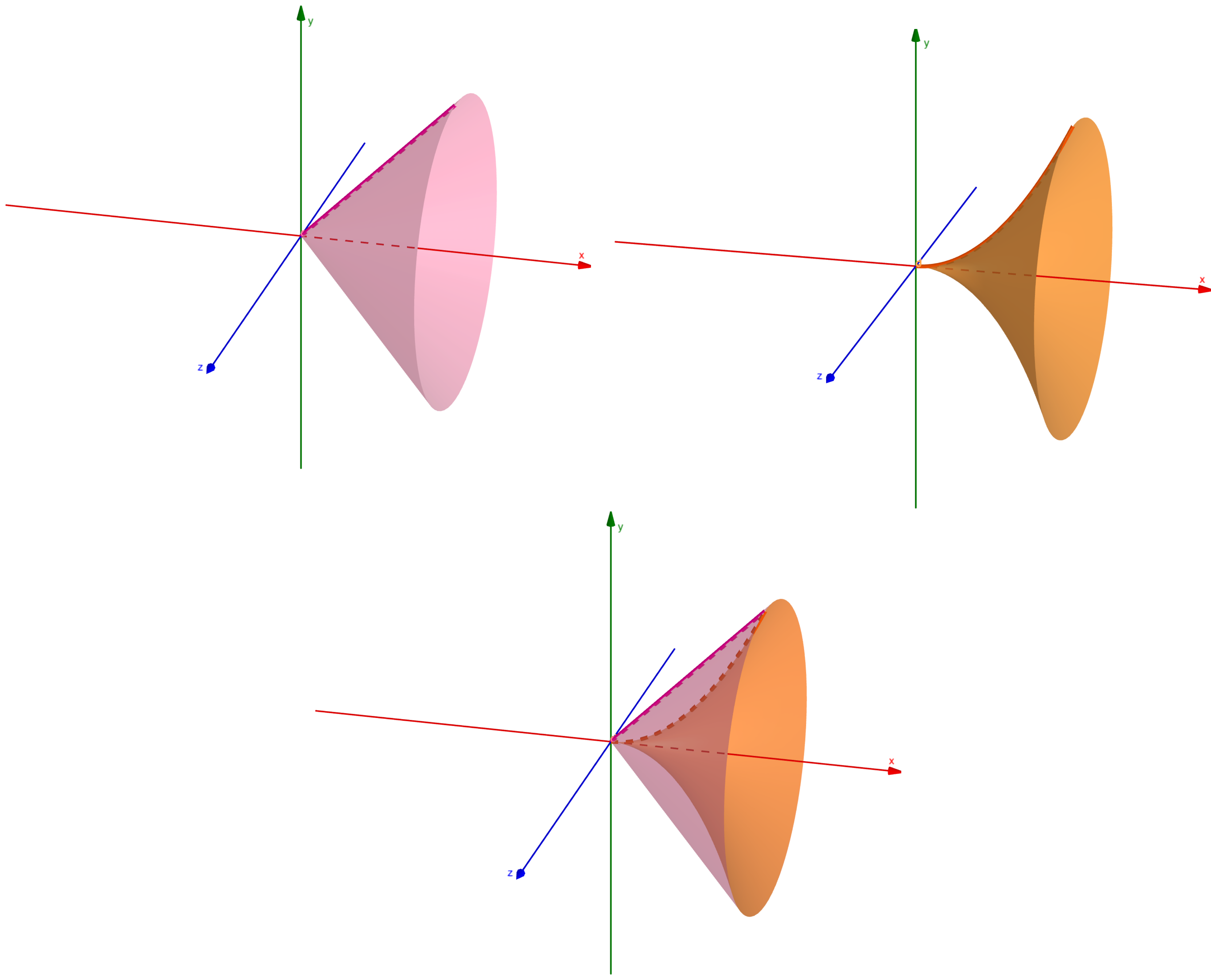
20. The diagram shows, the curve $\frac{1}{4}x^2$ intersects the straight line $y = x$ at O and $A(4, 4)$. Find

(a) the generated volume, in terms of π, when the shaded region is revolved fully about the y-axis.

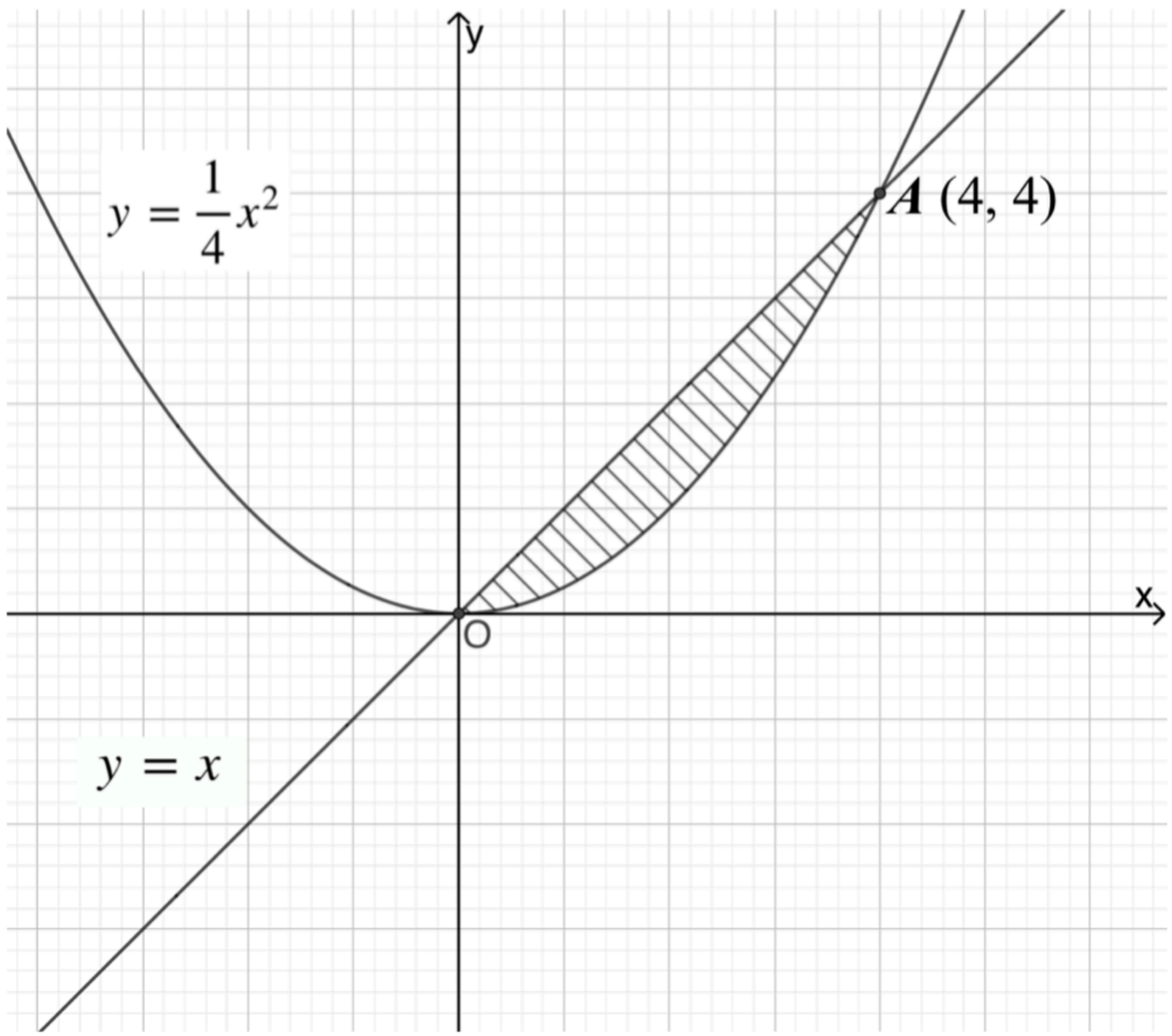
(b) the generated volume, in terms of π, when the shaded region is revolved fully about the x-axis.

b) Generated Volume = Volume Cone − Volume by Curve





(b) the generated volume, in terms of π, when the shaded region is revolved fully about the x-axis.



b) *Generated Volume = Volume Cone − Volume by Curve*

$$= \frac{1}{3}\pi r^2 h - \int_0^4 \pi y^2 \, dx$$

$$= \frac{1}{3}\pi(4^2)(4) - \left[\pi \int_0^4 \left(\frac{x^2}{4}\right)^2 dx\right]$$

$$= \frac{64}{3}\pi - \left(\frac{\pi}{16}\int_0^4 x^4 \, dx\right)$$

$$= \frac{64}{3}\pi - \left(\frac{\pi}{16}\left[\frac{x^5}{5}\right]_0^4\right)$$

$$= \frac{64}{3}\pi - \left[\frac{\pi}{16}\left(\frac{(4)^5}{5} - 0\right)\right]$$

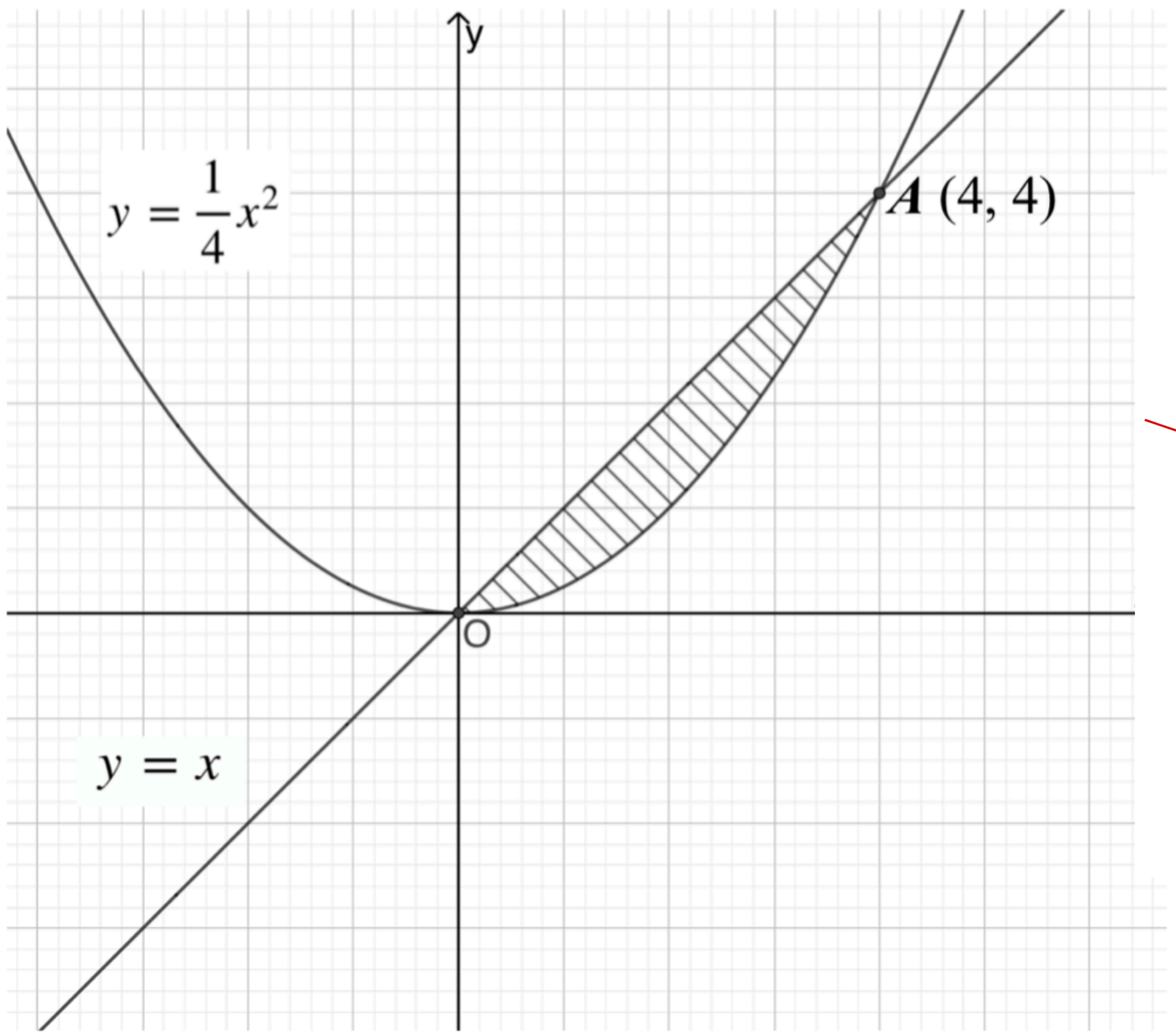
$$= \frac{64}{3}\pi - \frac{\pi}{16}\left(\frac{1024}{5}\right)$$

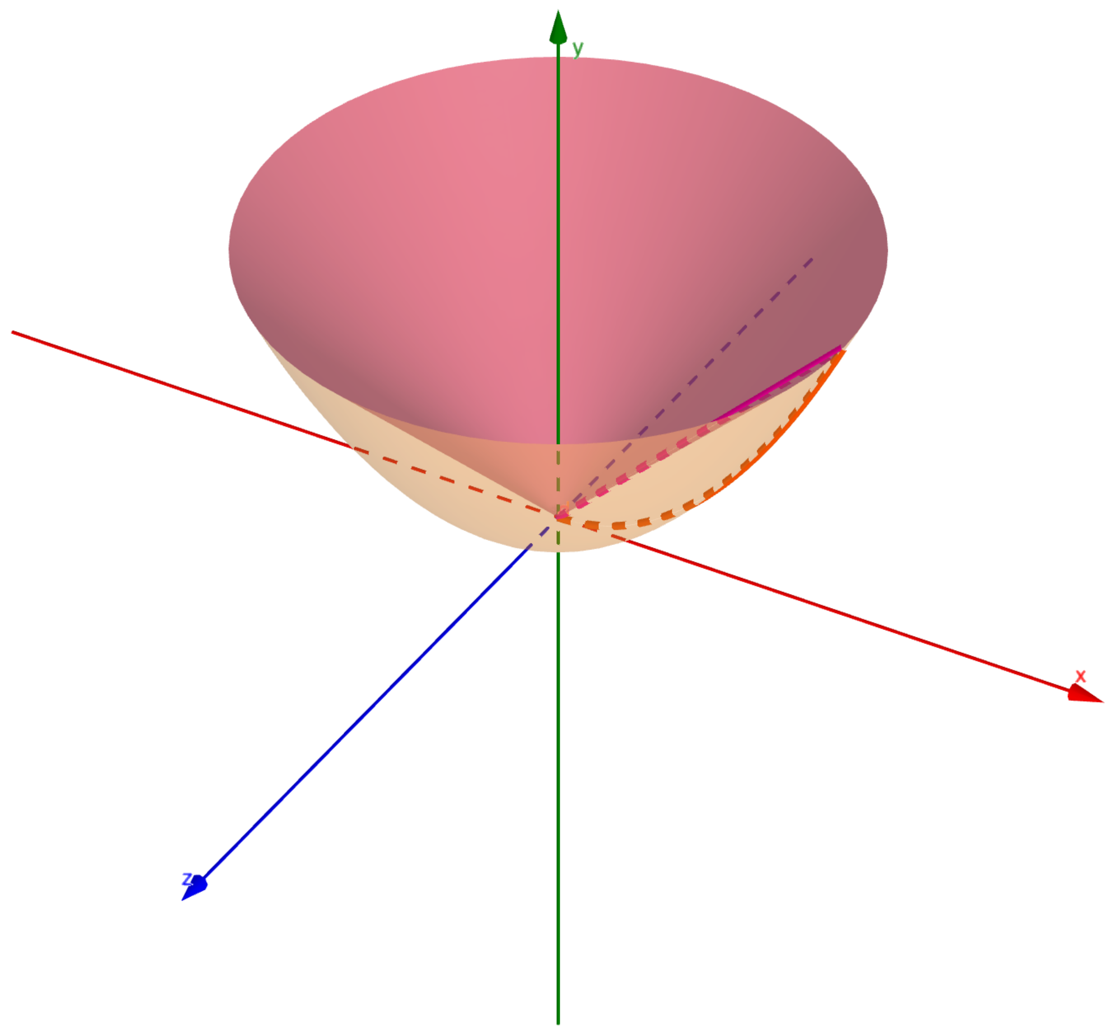
$$= \frac{64}{3}\pi - \frac{64}{5}\pi \quad = \frac{128}{15}\pi$$

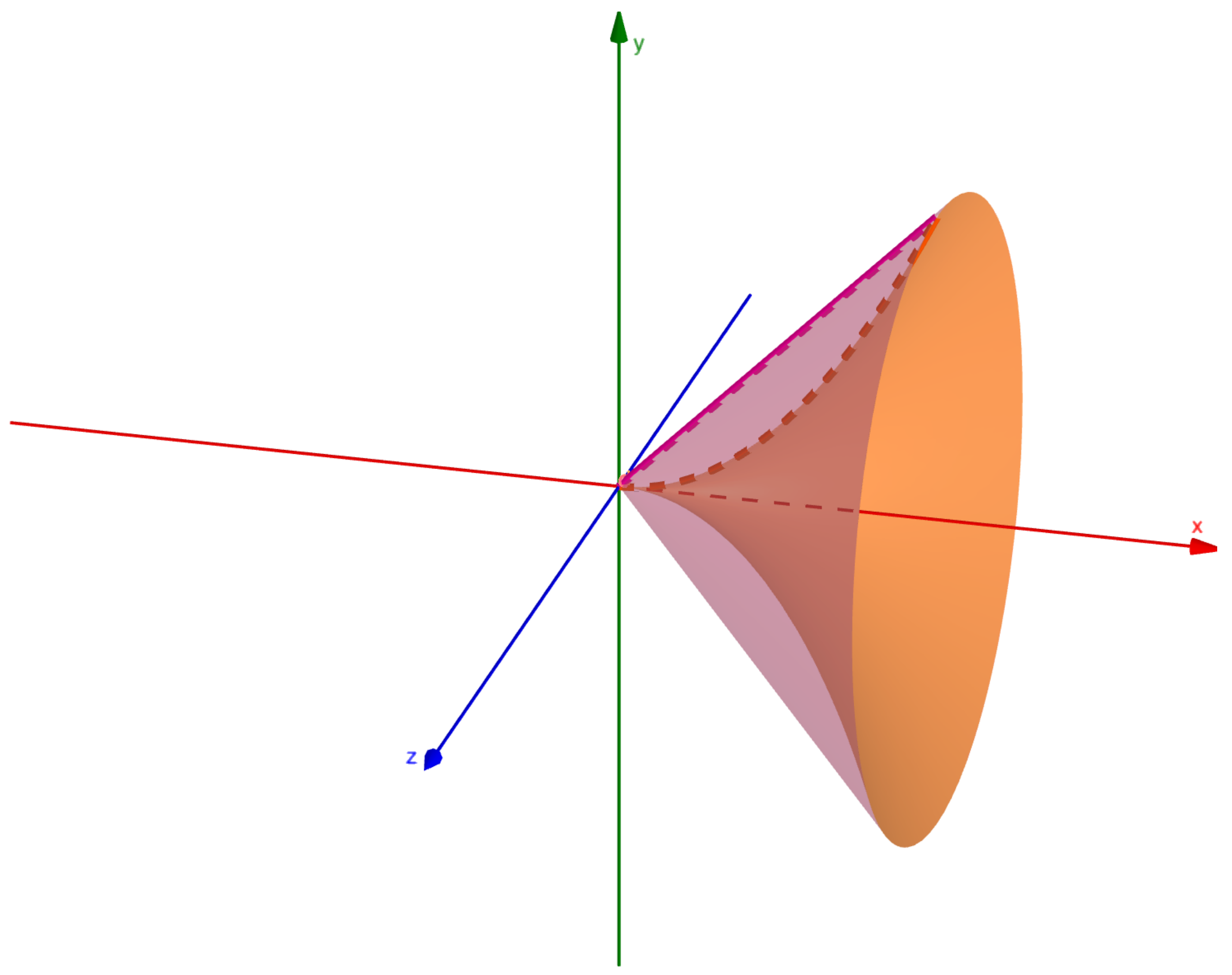
20. The diagram shows, the curve $\frac{1}{4}x^2$ intersects the straight line $y = x$ at O and $A(4, 4)$. Find

(a) the generated volume, in terms of π, when the shaded region is revolved fully about the y-axis.

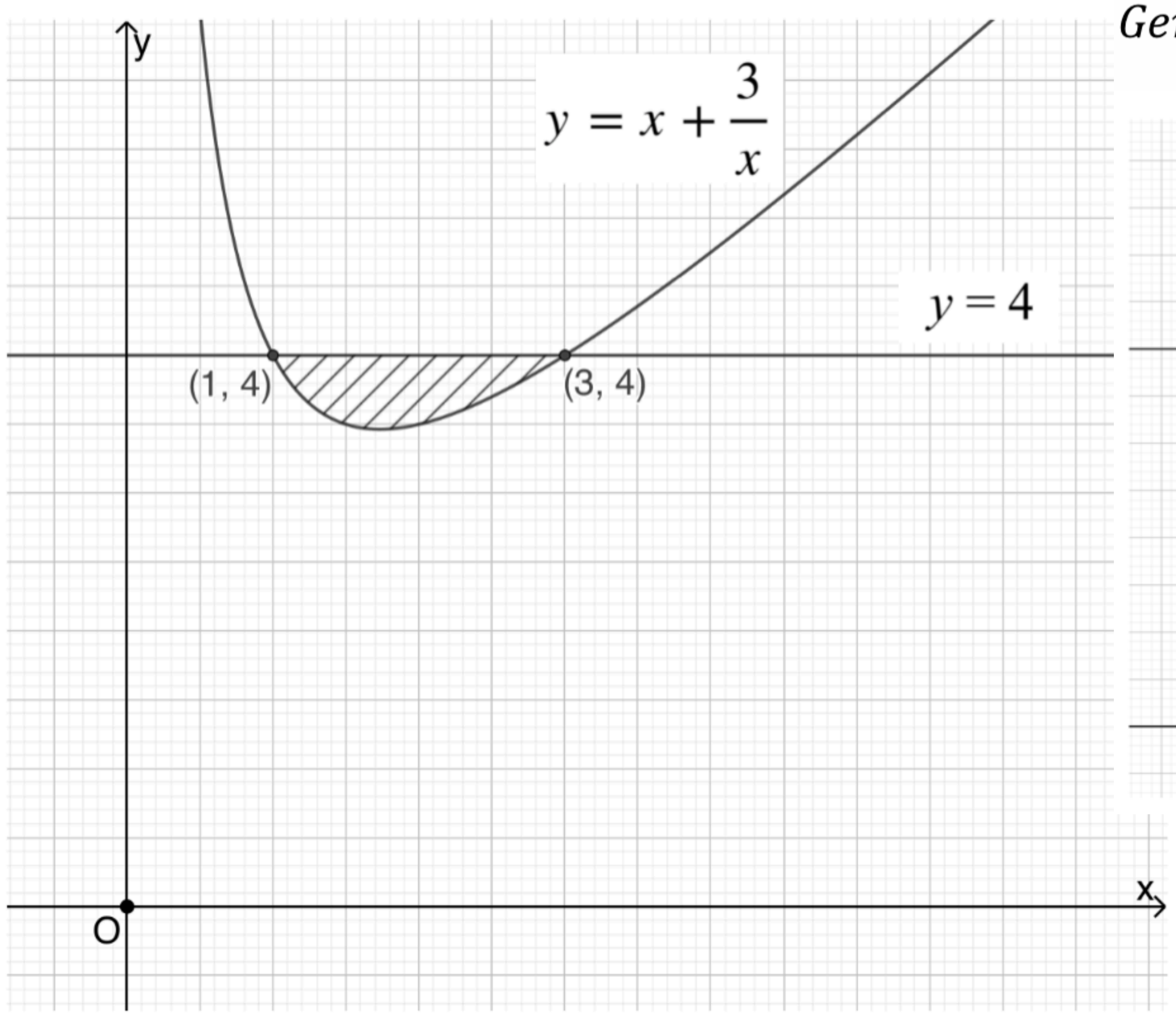
(b) the generated volume, in terms of π, when the shaded region is revolved fully about the x-axis.



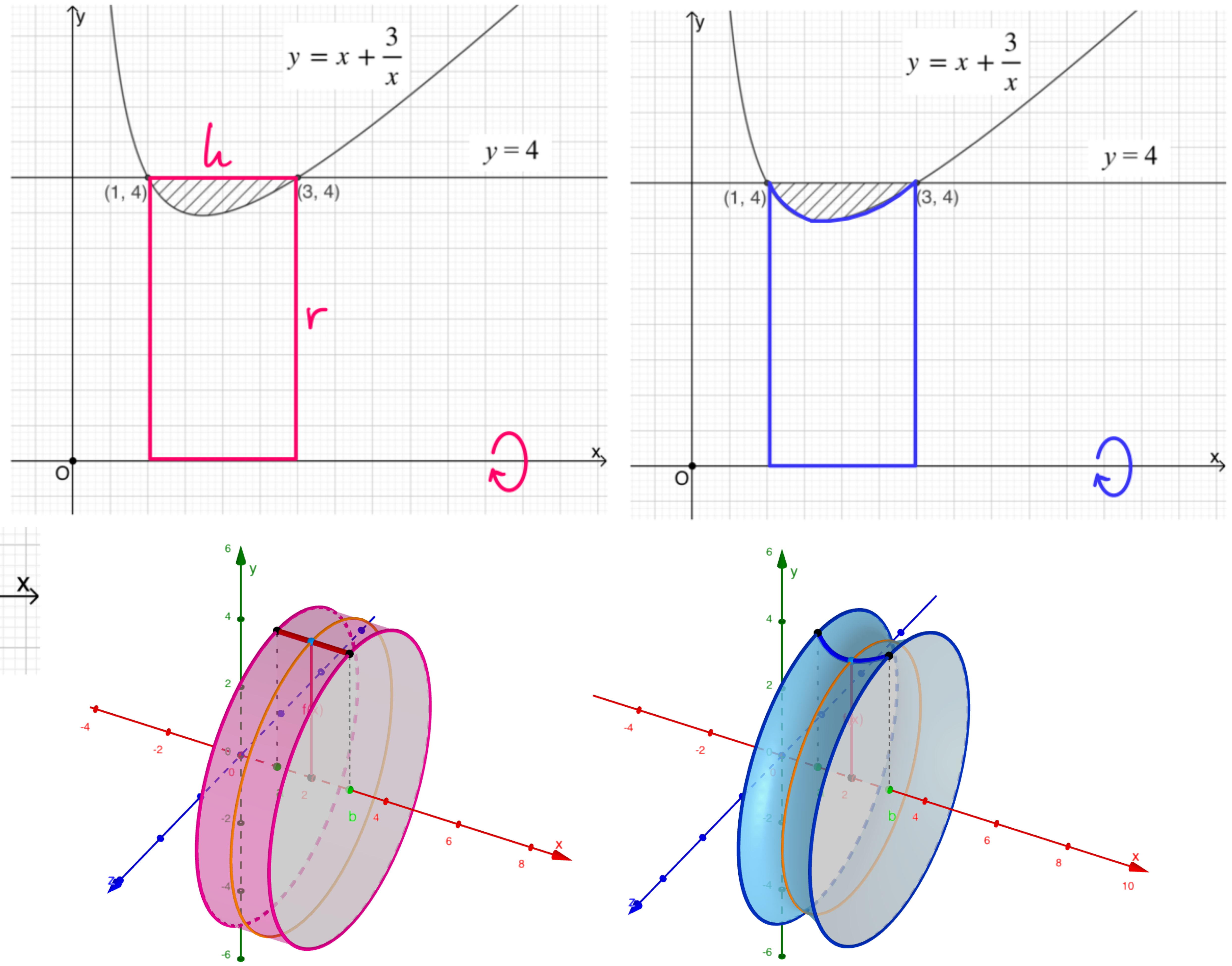




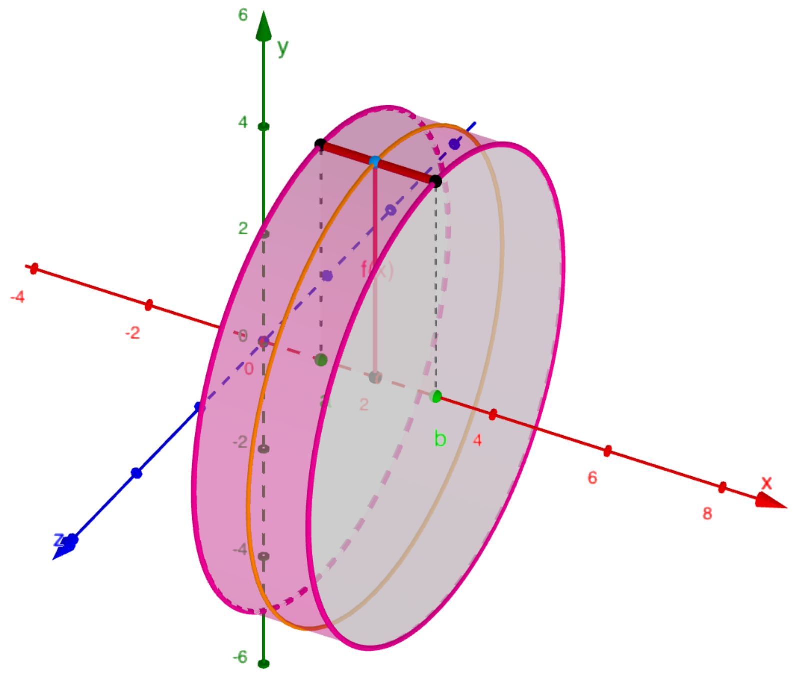
21. The diagram shows part of the curve $y = x + \dfrac{3}{x}$. and the straight line $y = 4$ which intersect at the points $P(1, 4)$ and $Q(3, 4)$. Find the volume generated when the shaded region is revolved through 360° about the x-axis.

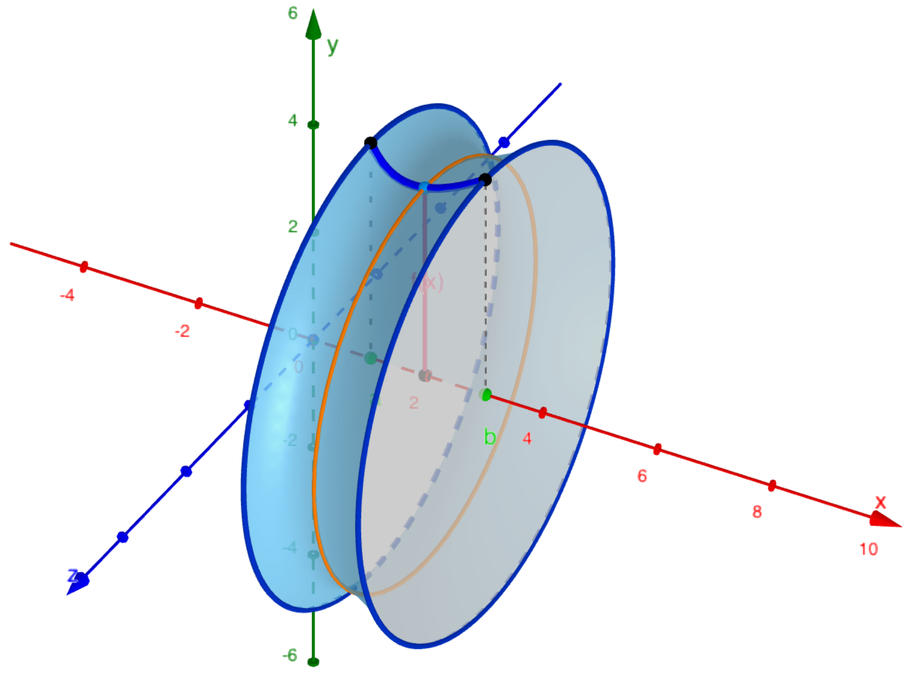


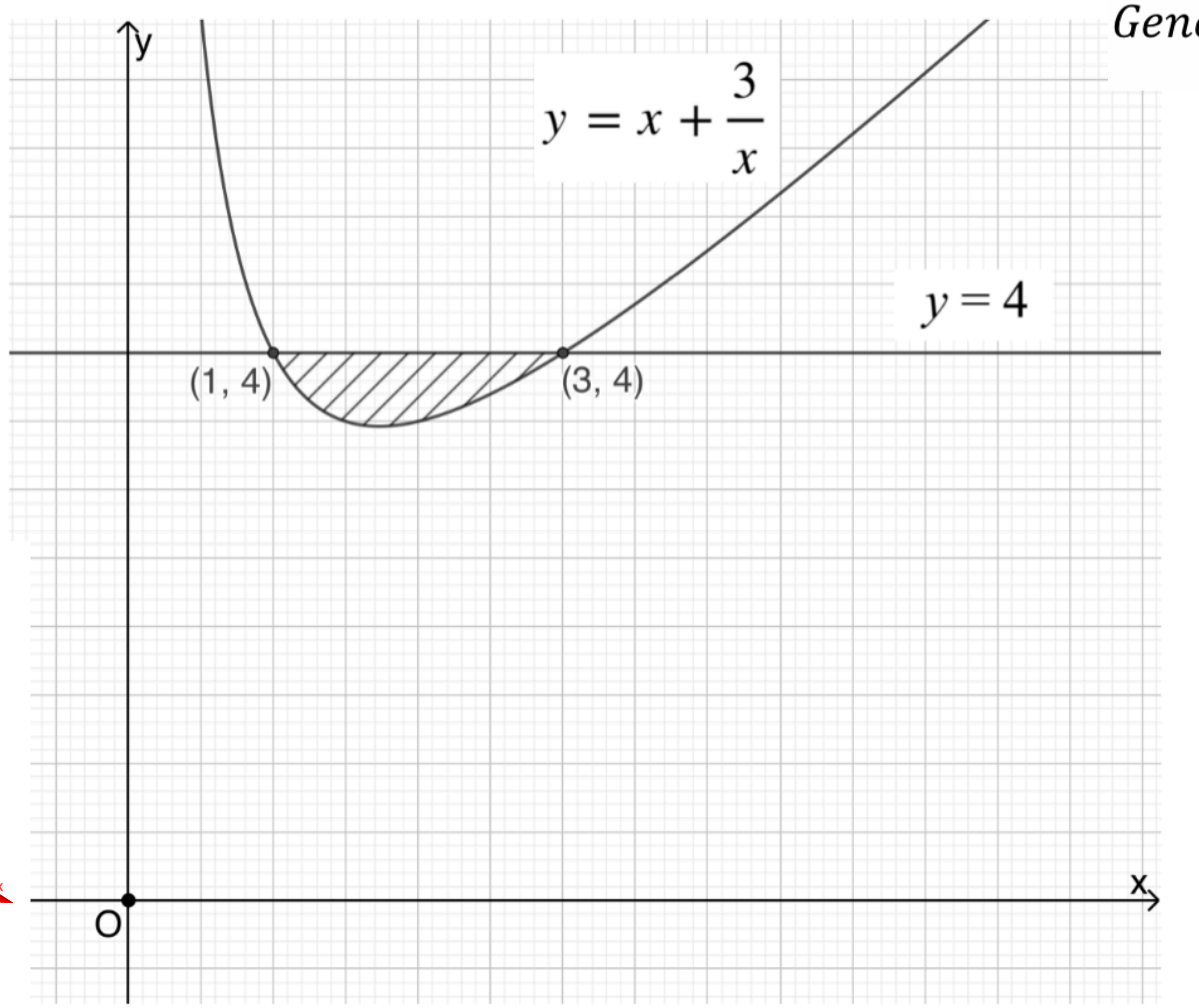
$Generated\ Volume = Vol.\ of\ "perfect"\ Cylinder - Vol.\ of\ "dented"\ Cylinder$



21. The diagram shows part of the curve $y = x + \frac{3}{x}$. and the straight line $y = 4$ which intersect at the points $P(1, 4)$ and $Q(3, 4)$. Find the volume generated when the shaded region is revolved through 360° about the x-axis.







$$y = x + \frac{3}{x}$$

$$y^2 = \left(x + \frac{3}{x}\right)^2$$

$$\therefore y^2 = x^2 + 6 + 9x^{-2}$$

Generated Volume = Vol. of "perfect" Cylinder − Vol. of "dented" Cylinder

$$= \pi r^2 h - \pi \int_1^3 y^2 \, dx$$

$$= \pi(4^2)(2) - \left(\pi \int_1^3 (x^2 + 6 + 9x^{-2}) \, dx\right)$$

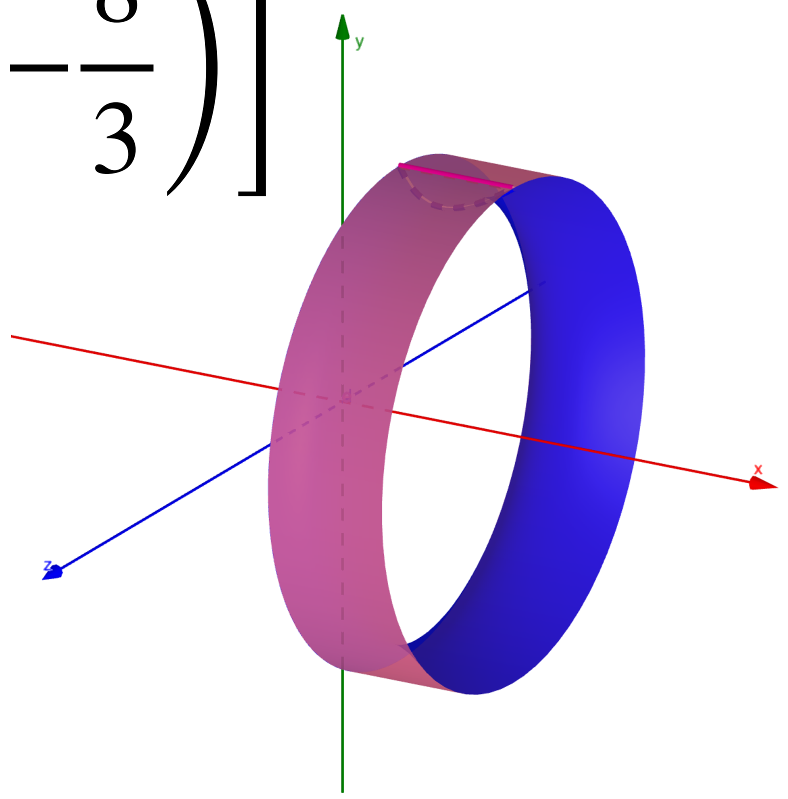
$$= 32\pi - \left(\pi \left[\frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1}\right]_1^3\right)$$

$$= 32\pi - \pi\left[(9 + 18 - 3) - \left(\frac{1}{3} + 6 - 9\right)\right]$$

$$= 32\pi - \pi\left[24 - \left(-\frac{8}{3}\right)\right]$$

$$= 32\pi - \frac{80}{3}\pi$$

$$= \frac{16}{3}\pi$$



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Part 2

Selected Past year Questions

Diagram 5 shows a straight line DP which is normal to the curve at point $P(4, 12)$.

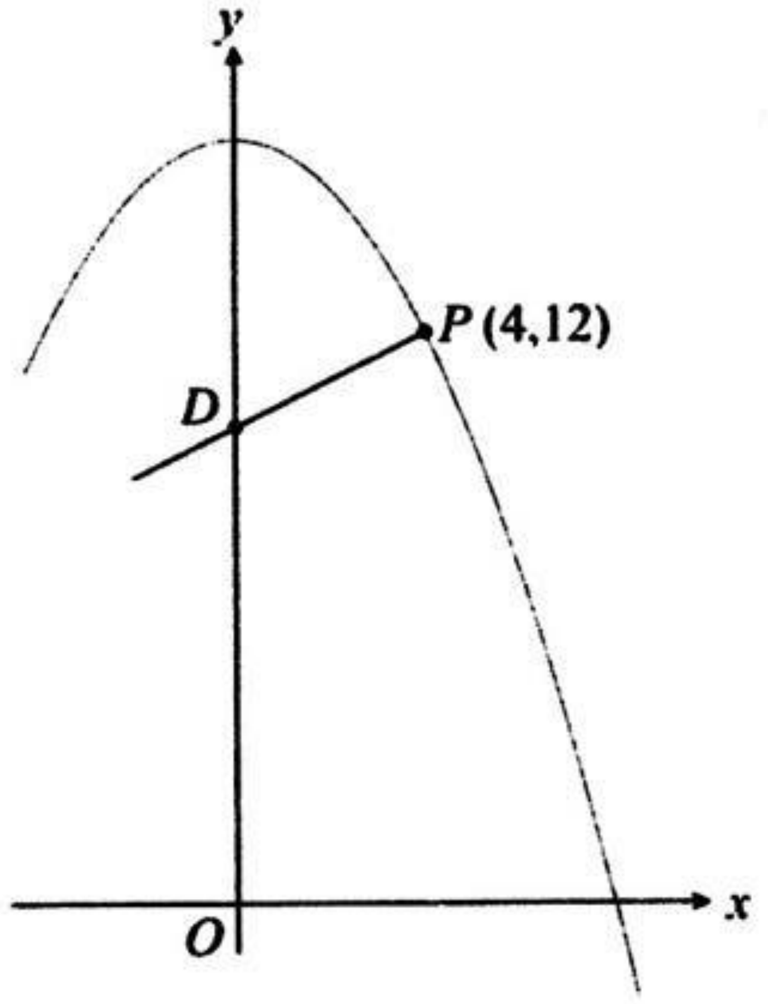


Diagram 5

The gradient function of the curve is $-\frac{1}{2}x$.

(*a*) Find

(i) the y-coordinate of point D,

(ii) the equation of the curve.

[6 *marks*]

(*b*) The volume generated when the region bounded by the curve, the y-axis and the straight line $y = k$ is revolved through 360° about the y-axis is 50π unit^3. Find the value of k.

[4 *marks*]

22. SPMRSM 2019 (no. 8/P2)

$$(a)\ \frac{dy}{dx} = -\frac{1}{2}x$$

$$At\ (4\ ,\ 12)\ ;$$

$$\frac{dy}{dx} = -\frac{1}{2}(4)$$

$$= -2$$

$$M_N \times M_t = -1$$

$$M_N \times -2 = -1$$

$$M_N = \frac{1}{2}$$

$$i)\ D(0,y)$$

$$\frac{y-12}{0-4} = \frac{1}{2}$$

$$2y - 24 = -4$$

$$2y = 20$$

$$y = 10$$

$$\therefore y\ -coordinate\ = 10$$

Diagram 5 shows a straight line DP which is normal to the curve at point $P(4, 12)$.

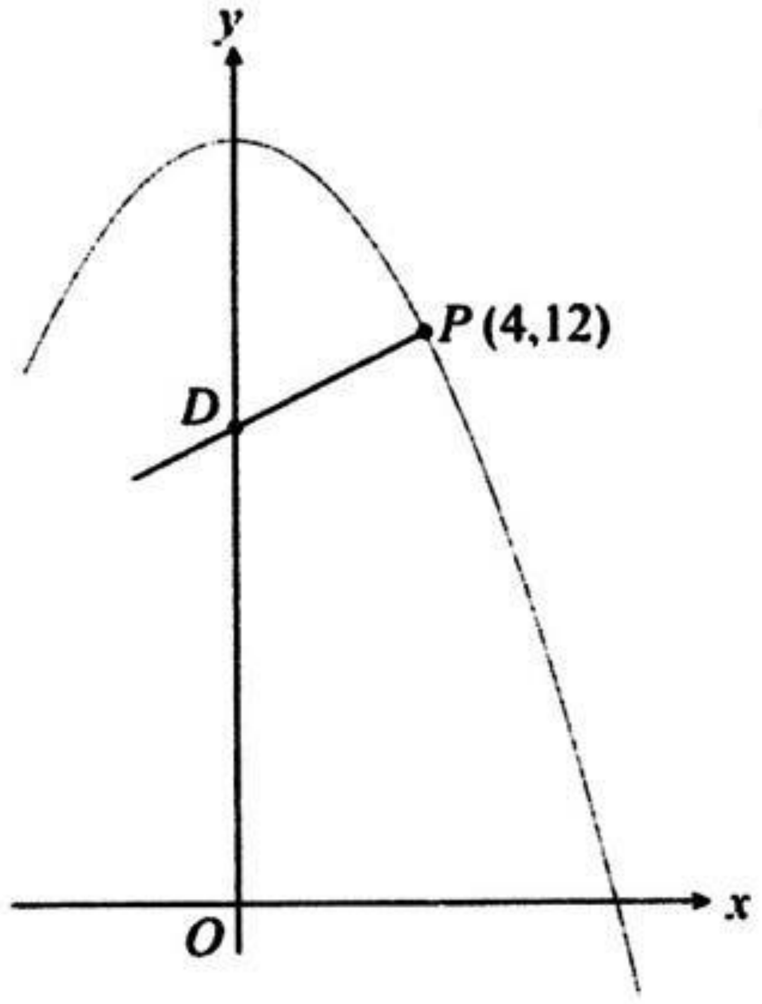


Diagram 5

The gradient function of the curve is $-\frac{1}{2}x$.

(*a*) Find

(i) the y-coordinate of point D,

(ii) the equation of the curve.

[6 *marks*]

(*b*) The volume generated when the region bounded by the curve, the y-axis and the straight line $y = k$ is revolved through $360°$ about the y-axis is 50π unit³.
Find the value of k. [4 *marks*]

22. SPMRSM 2019 (no. 8/P2)

(a) (ii) equation of the curve

$$\frac{dy}{dx} = -\frac{1}{2}x$$

$$y = \int -\frac{x}{2}\,dx$$

$$y = -\frac{1}{2}\int x\,dx$$

$$y = -\frac{1}{2}\left(\frac{x^2}{2}\right) + c$$

$$y = -\frac{1}{4}x^2 + c \rightarrow \;;\; P(4,12)$$

$$12 = -\frac{1}{4}(4)^2 + c$$

$$c = 16$$

$$\therefore y = -\frac{x^2}{4} + 16$$

Diagram 5 shows a straight line *DP* which is normal to the curve at point $P(4, 12)$.

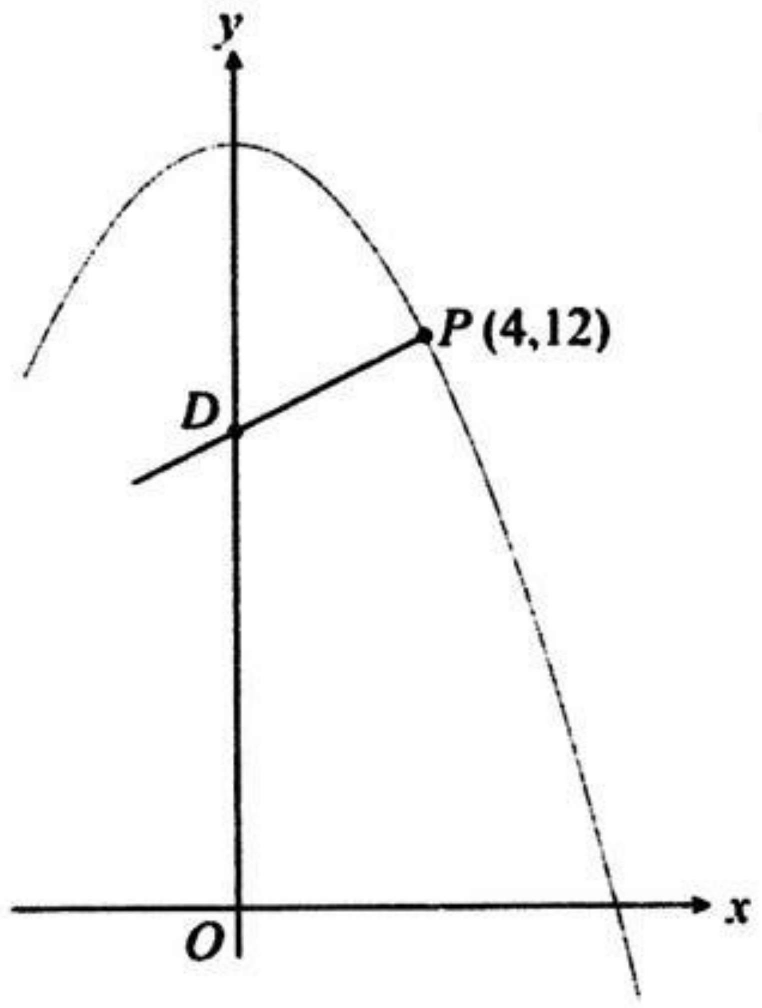
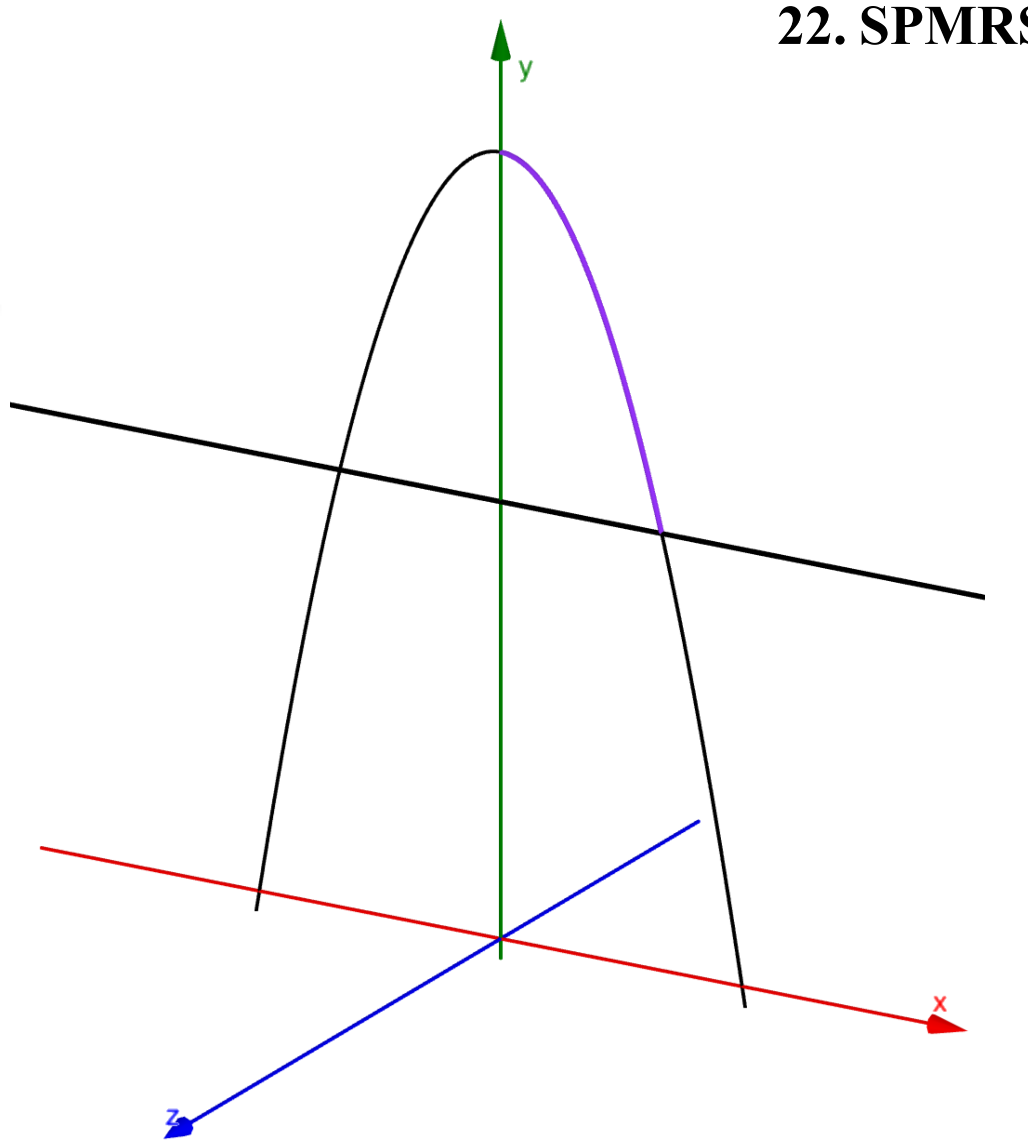
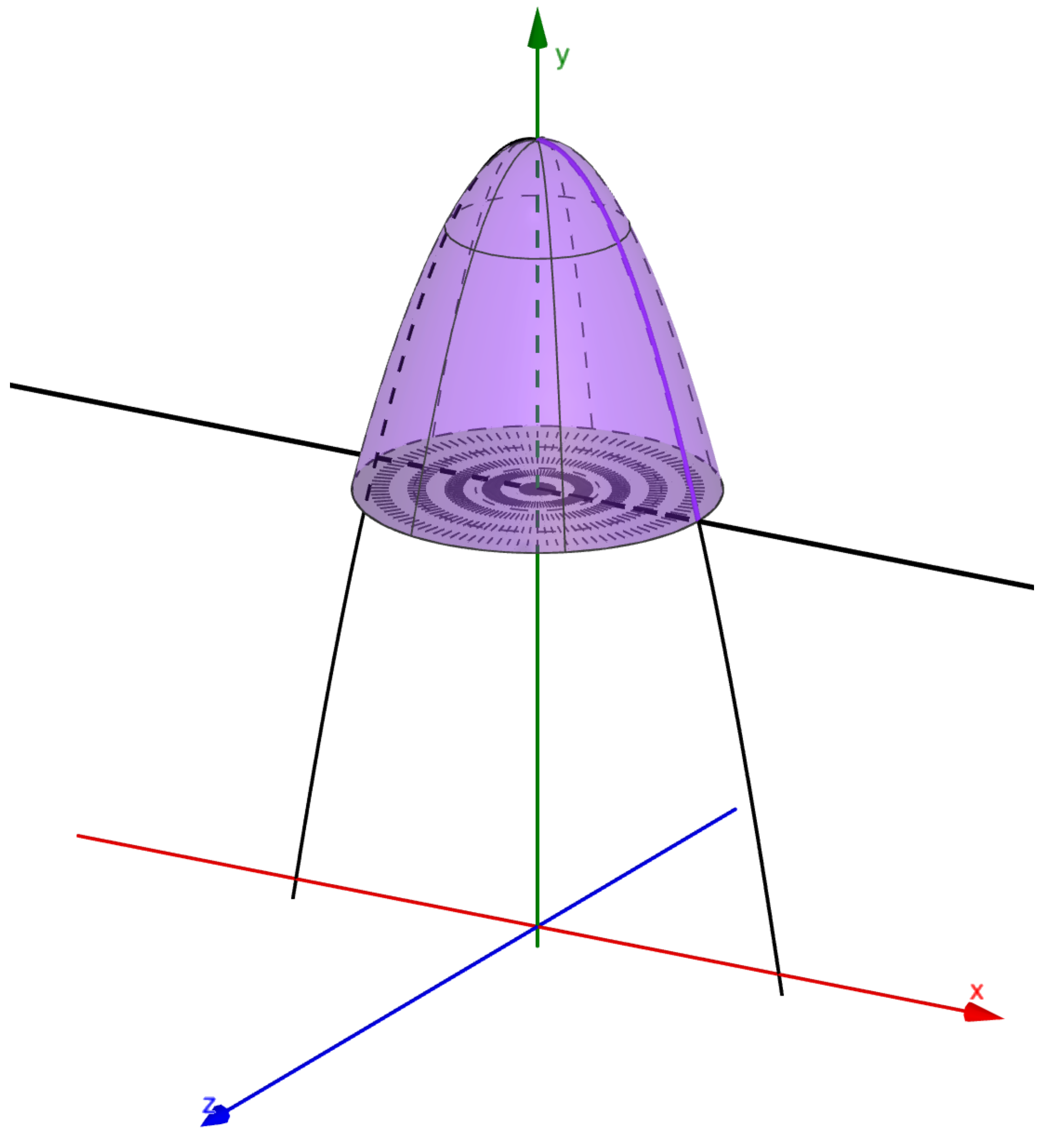


Diagram 5





The gradient function of the curve is $-\frac{1}{2}x$.

(*a*) Find

(i) the *y*-coordinate of point *D*,

(ii) the equation of the curve.

[6 *marks*]

(*b*) The volume generated when the region bounded by the curve, the *y*-axis and the straight line $y = k$ is revolved through 360° about the *y*-axis is 50π unit³. Find the value of *k*. [4 *marks*]

$$b)\ V = \int_k^{16} \pi x^2\, dy$$

$$y = -\frac{x^2}{4} + 16$$

$$x^2 = 64 - 4y$$

$$50\pi = \pi \int_k^{16} (64 - 4y)\ dy$$

$$50 = \left[64y - 2y^2\right]_k^{16}$$

$$50 = \left[1024 - 2(256)\right] - \left[64k - 2k^2\right]$$

$$50 = 512 - 64k + 2k^2$$

$$2k^2 - 64k + 462 = 0$$

$$k^2 - 32k + 231 = 0$$

$$(k - 21)(k - 11) = 0$$

$$k = 21\ ,\ k = 11$$

$$\therefore k = 11$$

Diagram 2 shows the curve $y = 4x - x^2$ and tangent to the curve at point Q passes point P.

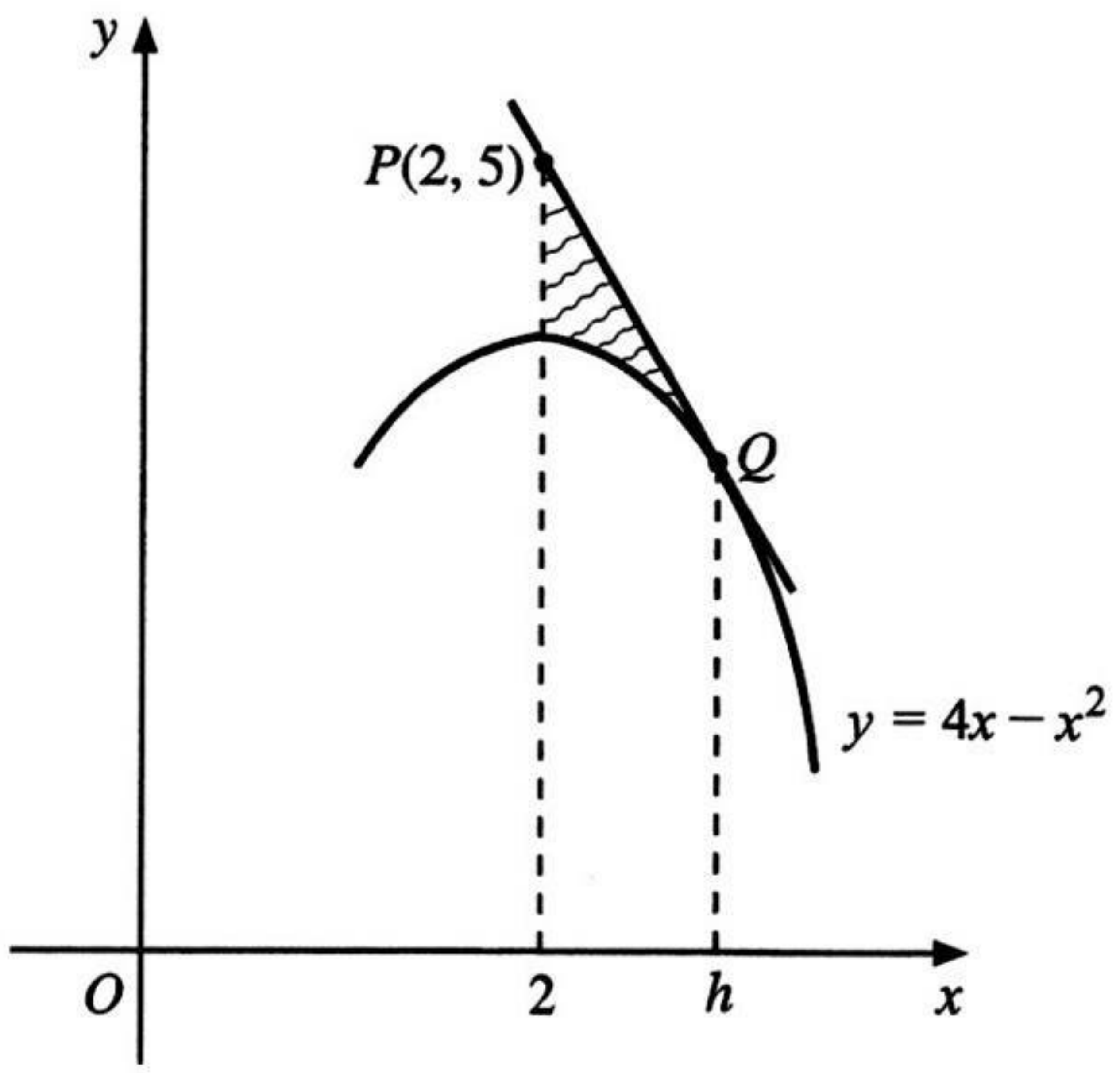


Diagram 2

(a) Show that $h = 3$. [4 marks]

(b) Calculate the area of the shaded region. [4 marks]

a) $y = 4x - x^2$

$$\frac{dy}{dx} = 4 - 2x$$

At point $Q(h\ ,\ 4h - h^2)$

$$\frac{dy}{dx} = 4 - 2h$$

Equation of tangent at $Q(h\ ,\ 4h - h^2)$

$$y - y_1 = m(x - x_1)$$

$$y - (4h - h^2) = (4 - 2h)(x - h)$$

$\because$ tangent passes through $P(2,\ 5)$

$$\therefore 5 - (4h - h^2) = (4 - 2h)(2 - h)$$

$$5 - 4h + h^2 = 8 - 4h - 4h + 2h^2$$

$$h^2 - 4h + 3 = 0$$

$$(h - 1)(h - 3) = 0$$

$$h = 1\ ,\ h = 3$$

$$\therefore\ h = 3$$

Diagram 2 shows the curve $y = 4x - x^2$ and tangent to the curve at point Q passes point P.

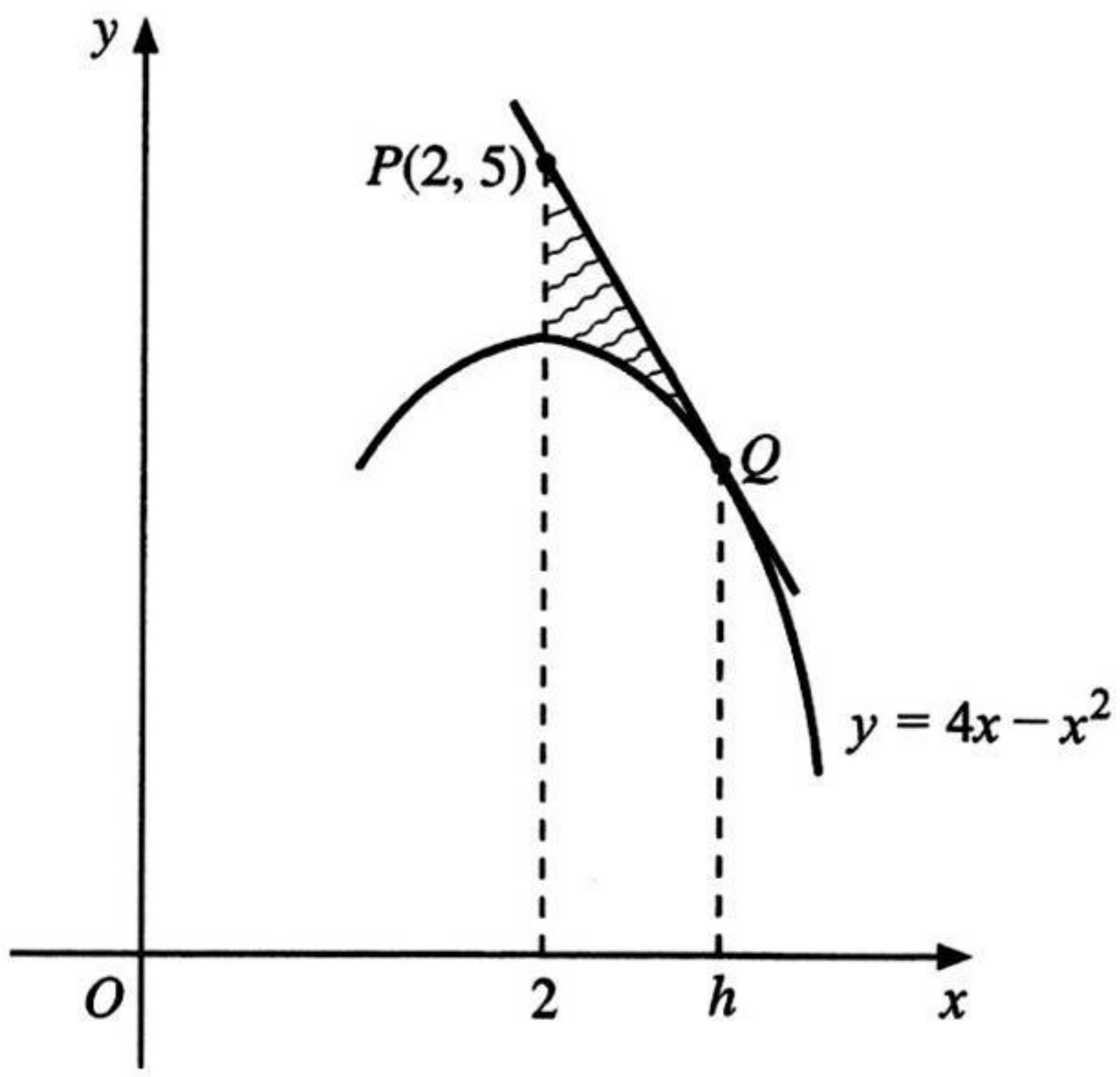


Diagram 2

(*a*) Show that $h = 3$. [4 *marks*]

(*b*) Calculate the area of the shaded region. [4 *marks*]

$b)$ *Area of Shaded region*

$$= Area\ of\ Trapezium - Area\ under\ curve$$

$$= \left[\frac{1}{2} \times \left(5 + 4h - h^2\right) \times 1\right] - \int_2^3 y\, dx$$

$$= \left[\frac{1}{2} \times \left(5 + 3\right)\right] - \int_2^3 \left(4x - x^2\right)\ dx$$

$$= 4 - \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_2^3$$

$$= 4 - \left[\left(2(9) - 9\right) - \left(2(4) - \frac{8}{3}\right)\right]$$

$$= 4 - \left[9 - \frac{16}{3}\right]$$

$$= 4 - \frac{11}{3}$$

$$= \frac{1}{3}$$

SELAMAT MAJU JAYA

Terima Kasih

Mohamad Fauzi Razak | 18 October 2021


Siri Jom Skor A+ Matematik Tambahan
SPM 2021
22 Aug
8.00 pm – 10.00 pm
Sahlawati Zakaria | MRSM Kuala Krai
Functions
27 Aug
8.00 pm – 10.00 pm
Norlela Sapari | MRSM Taiping
Quadratic Functions
31 Aug
8.00 pm – 10.00 pm
Khairulbariah Khairuddin | MRSM Mersing
Systems of Equations
4 Sept
8.00 pm – 10.00 pm
Hazlina Ahmad | MRSM Alor Gajah
Indices, Surds and Logarithms
10 Sept
8.00 pm – 10.00 pm
Hasniza Ismail | MRSM Parit
Progressions
16 Sept
3.00 pm – 5.00 pm
Rosdiana Sarju | MRSM Johor Bahru
Linear Law
24 Sept
8.00 pm – 10.00 pm
Nur Suhaila Abu Bakar | MRSM Tumpat
Coordinate Geometry
New Speaker
26 Sept
8.00 pm – 10.00 pm
Mohd Faizi Mamat | MRSM Gemencheh
Vectors
1 Oct
8.00 pm – 10.00 pm
Abdul Hadi Azmi | MRSM Pengkalan Chepa
Solution of Triangles
8 Oct
8.00 pm – 10.00 pm
Noraini Ismail | MRSM Transkrian
Index Numbers
10 Oct
8.00 pm – 10.00 pm
Hariani Abidin | MRSM Kuching
Circular Measure
15 Oct
8.00 pm – 10.00 pm
Erwan Hazreen Musa | MRSM Bentong
Differentiation
18 Oct
8.00 pm – 10.00 pm
Mohamad Fauzi Razak | MRSM Kepala Batas
Integration
New Date
23 Oct
8.00 pm – 10.00 pm
Muhamad Baginda Zainuddin | MRSM Batu Pahat
Kinematics of Linear Motion
New Date
30 Oct
3.00 pm – 5.00 pm
Haziq Syazwan Sajali | MRSM Tun Mustapha
Trigonometric Function
New Date
5 Nov
3.00 pm – 5.00 pm
Suhaila Sulong | MRSM Tun Dr. Ismail
Permutation and Combination
New Date
7 Nov
8.00 pm – 10.00 pm
Norhafizah Mohamed Yusoff | MRSM K. Terengganu
Probability Distribution
New Date
17 Dis
8.00 pm – 10.00 pm
Asniza Arshad | MRSM Tun Ghaffar Baba
Linear Programming
Anjuran Unit Matematik
Bahagian Pendidikan Menengah MARA
Sesi webinar live melalui Microsoft Teams